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Discussion Paper #2022.04

# Market Design for Distributional Objectives in (Re)assignment: An Application to Improve the Distribution of Teachers in Schools

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**January 2025**

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**Abstract**

Centralized (re)assignment of workers to jobs is increasingly common in public and private sectors. However, these markets often suffer from distributional problems. We propose a new strategy-proof mechanism that efficiently improves individual and distributional welfare over the status quo. We justify our constructive and practical approach by micro-founding it through the theory of inequality measures in welfare economics. To evaluate the performance of our mechanism, we focus on teacher (re)assignment, where the unequal distribution of experienced teachers across schools is a well-documented concern. Using French data, we demonstrate that our mechanism reduces the teacher experience gap across regions more effectively than benchmarks, including the current mechanism, while providing higher average welfare for teachers.

**JEL codes**: C78, D50, D61, D67, I21

**Keywords**: Matching Theory, Market Design, Priority Design, Teacher Reassignment, Status-quo Improvement, Efficiency, Inequality Measures, Distributional Welfare

# 1 Introduction

The centralized (re)assignment of workers, which involves the initial assignment of new employees and the reassignment of existing ones to jobs, tasks, or managers, is in-

*First Draft: February 2022. We sincerely thank the Coeditor, Pietro Ortoleva, and three anonymous referees for their comments, which significantly enhanced the quality of this paper. We also thank Bobak Pakzad-Hurson, Parag A. Pathak, and the audience members and discussants at Brown, 2024 CMID, Stony Brook, Rice Paris Market Design Conference, Lausanne Market Design Workshop, Concordia CIREQ, Karlsruhe IT, Tinbergen, CREST Mannheim Workshop, Manchester, 2021 NBER Market Design Working Group Meeting, Rochester, 2021 Decentralization Conference, 2021 ASSA Meetings, Tokyo, Deakin, and Boğaziçi for comments. Combe: Department of Economics, École Polytechnique julien.combe@polytechnique.edu; Dur: Department of Economics, North Carolina State University umutdur@gmail.com; Tercieux: Paris School of Economics and CNRS, tercieux@pse.ens.fr; Terrier: Department of Economics, Queen Mary University London c.terrier@qmul.ac.uk; Ünver: Department of Economics, Boston College and Professorial Research Fellow, Deakin University unver@bc.edu

creasingly common in both the public and private sectors. In many countries, doctors are centrally (re)assigned to hospitals (Kamada and Kojima, 2015), police officers to precincts (Sidibe et al., 2021), teachers to public schools (Dur and Kesten, 2019, Combe et al., 2022, Bobba et al., 2021, Bates et al., 2021), and civil servants to regional jobs (Thakur, 2020). Centralized (re)assignment also exists within large corporations, where rotation procedures are commonly used to (re)assign workers to jobs (Cheraskin and Campion, 1996).

From a market design perspective, these labor markets differ from other matching markets studied in the literature. First, they are characterized by the presence of both *new workers*, who require an initial job assignment, and *existing workers*, who vacate their current positions during the (re)assignment process. As a result, there are both already-occupied and vacant positions to consider. Second, for existing workers, participation is usually voluntary. If they do participate, reassignment occurs only when a better position than their current one becomes available. This built-in property right can conflict with employers' objectives, such as achieving a balanced distribution of workers across jobs. Market designers must carefully address these novel aspects.

The centralized (re)assignment of workers is also significant from a policy and design perspective because many of these markets suffer from distributional challenges.[1] Some countries and cities attempt to address these disparities by offering higher salaries or better working conditions in less desirable jobs or locations (Bobba et al., 2021, Biasi, 2021, Falch, 2010). However, these strategies face two key constraints: public-sector salaries for civil servants, such as teachers, doctors, and police officers, are often tied to rigid pay schedules, limiting their effectiveness as incentives. Additionally, in professions driven by intrinsic motivation, low wage elasticity causes these policies to be either ineffective or prohibitively costly (Bobba et al., 2021, Bates et al., 2021). In such contexts, centralized (re)assignment offers an opportunity to leverage novel mechanism design to mitigate distributional challenges.

This paper is motivated by the following research question: In the complex environments described, how can we design (re)assignment mechanisms that achieve specific distributional objectives? To address this question, we introduce a new model and an assignment mechanism rooted in the theory of inequality measures from welfare economics. This mechanism is designed to achieve a more desirable worker distribution as determined by policymakers. We then empirically assess the extent to which the new mechanism improves worker distribution.

As our empirical application, we study the (re)assignment of public school teach-

[1] For example, rural hospitals struggle to recruit doctors (Kamada and Kojima, 2015). Police officers tend to avoid urban city centers prone to violence (Sidibe et al., 2021). Public administrators in India prefer and are often assigned jobs near their home states, which poses obstacles to national integration objectives (Thakur, 2020). Moreover, high-quality teachers are rarely distributed equitably among schools (Hanushek et al., 2004, Jackson, 2009).

ers in France, a market that suffers from significant and persistent distributional problems. Globally, good teachers tend to work in schools serving more affluent students and schools with a higher share of native and high-achieving students (Bobba et al., 2021, Bates et al., 2021, Biasi, 2021, Hanushek et al., 2004, Jackson, 2009, Bonesrønning et al., 2005, Allen et al., 2018). Despite a growing body of research examining the effectiveness of wage policies in attracting high-quality teachers to disadvantaged schools in decentralized hiring systems, our understanding of the role played by mechanisms in centralized (re)assignment systems remains limited.[2] This paper aims to address this gap. While we present the theory with a focus on teachers, all theoretical results apply more broadly to other applications.[3]

**Model and design desiderata.** We introduce a model in which new teachers seek their first assignments, and tenured teachers seek reassignments. Tenured teachers already have initial jobs, represented by a *status-quo matching*. Each teacher has strict preferences over schools.[4] A policymaker manages the schools and determines the matching of teachers to schools.

We focus on two main matching properties. First, matchings should weakly *status-quo improve* the teacher distribution at each school. To achieve this, each teacher has a *type* capturing observable characteristics, such as experience, education, or past performance. The policymaker assigns a *type ranking* to each school. For example, type rankings might prioritize low-experience teachers in schools with many high-experience teachers and vice versa. We allow arbitrary type rankings to accommodate diverse objectives. A status-quo improvement is assessed through Lorenz domination, analogous to first-order stochastic dominance, comparing a given matching with the status-quo matching based on type rankings. In other words, a matching is status-quo improving for schools if, for each school and each type, the cumulative number of teachers with types that are weakly higher ranked is always larger under the new matching than under the status-quo matching.

Second, teachers' assignments should be *individually rational*, meaning they must be at least as good as their initial assignments. Together, these two properties—*individual ratio-*

[2]Countries that use a centralized process to assign teachers to schools include Germany, Czechia (Cechlárová et al., 2015), Italy (Barbieri et al., 2011), Türkiye (Dur and Kesten, 2019), Mexico (Pereyra, 2013), Peru (Bobba et al., 2021), Uruguay (Vegas et al., 2006), and Portugal.

[3]We discuss some of these applications in more detail in Appendix C.

[4] In our setting, teachers' preferences are primarily determined by exogenous factors, such as school location, rather than peers (see Appendix G). Furthermore, because the number of individuals seeking (re)assignment is small, our market is mostly immune to peer effects. For instance, in France in 2013, about 4.3% of secondary school teachers requested to change regions and about 2% additional were new teachers seeking first assignment (DEPP, 2014a). Thus, preferences over schools remain largely unaffected by peers, as most peers at schools remain unchanged. However, in other applications, preferences over peers may be critical, such as in police officer assignments (Sidibe et al., 2021). See Pycia (2012) for a theoretical treatment and Cox et al. (2021) for an empirical analysis in large markets.

*nality* and *status-quo improvement for schools*—ensure that enhancing teachers' welfare does not worsen the distribution of teachers, as measured by the policymaker's type rankings. As we will demonstrate, improving teacher distribution for a carefully constructed type ranking profile is closely linked to reducing inequality, provided a reasonable assumption about market size is satisfied.

In addition to individual rationality and status-quo improvement for schools, we aim to identify matchings that are Pareto undominated among those satisfying these two properties. We refer to such matchings as *SI-constrained efficient*. Since teacher preferences are private information, we further require SI-constrained efficient mechanisms to be *strategy-proof*. To achieve these goals, we introduce the *status-quo improving cycles and chains* (or *SI-CC*) mechanism, which satisfies these properties.

**The SI-CC mechanism.** The SI-CC mechanism is inspired by top-trading cycles (TTC) mechanisms, particularly those proposed by Shapley and Scarf (1974), Abdulkadiroğlu and Sönmez (1999), and Roth et al. (2004), but with a key distinction: unlike TTC-variant mechanisms, which ensure that only teachers are weakly better off compared to the status quo, the SI-CC mechanism guarantees that both teachers are weakly better off and the schools' teacher distribution weakly improves under the given type-ranking profile.[5]

The SI-CC outcome is determined through an iterative algorithm that operates on directed graphs. When a cycle or an appropriately defined chain in the directed graph is identified, each teacher within it is assigned to the school to which she points.

Compared to TTC algorithms, the SI-CC mechanism introduces two key innovations through the endogenous definition of pointing rules to construct the directed graphs. First, we define the *school pointing rule*, which determines the order in which a school allows its status-quo employees to be reassigned. By pointing, a school effectively grants permission for one of its status-quo employees to be assigned to a different school. The pointing order is constructed such that a school first points to its employee with the lowest type (who leaves first) and subsequently to employees with higher types.[6] Second, we define the *teacher pointing rule*, which specifies the schools to which a teacher may point and, consequently, be assigned. A teacher may point to a school if replacing her with the teacher pointed to by that school status-quo improves the school, or if there is a vacant position at the school that can be filled in a manner that status-quo improves the school. In the directed graph, a teacher points to the best school available to her under these conditions.

We carefully design the pointing rules and the order in which cycles and chains are resolved. Allowing substantial changes to these orderings could compromise SI-constrained

[5]This two-sided improvement feature of the SI-CC mechanism also relates it to the *efficient and stable* matching algorithm (Erdil and Ergin, 2017), which achieves Pareto-efficient stable matchings in two-sided markets with preference indifferences. See the literature review in Section 6 for further details.

[6]This order is practically the reverse of the type rankings that prioritize teachers.

efficiency under a type-ranking profile or strategy-proofness. However, our precise formulation ensures that the SI-CC mechanism satisfies both properties (Theorem 1).

**Inequality reduction and the design of type rankings.** An important property of the SI-CC mechanism is its simplicity and transparency, as it treats each school independently from the others and can be deployed in practice to replace similarly functioning mechanisms while improving teacher distribution. However, to understand how the policymaker's goal of reducing inequality in terms of teacher experience distribution across schools can be achieved through the SI-CC mechanism, we must address a critical question: under what type-ranking profiles (if any) does status-quo improvement lead to inequality reduction?

Even if such type-ranking profiles exist and lead to inequality reduction, the objective of inequality reduction is inherently interconnected with the eventual employee distribution across schools. A mechanism that violates status-quo improvement for schools might still achieve inequality reduction by making some schools worse off. However, we show that the status-quo improvement axiom for schools is not only sufficient but also necessary to achieve inequality reduction in large markets under certain smoothness and genericity assumptions. To formalize this, we microfound the policymaker's design of type rankings using the theory of inequality measures.

Consider a standard inequality index, such as the Gini (1912) index, the Atkinson (1970) index, or the ratio of the average experience of the top 20% of schools to that of the bottom 20% (referred to as the T20/B20 ratio index[7]), weighted by the population of teachers in each school. The arguments of the index are statistics such as the mean or median teacher experience in each school or the fractions of teachers exceeding a threshold level of experience.

We demonstrate that if the proportion of teachers who do not apply for annual reassignment is large relative to those who participate in each school, then there exists a *natural type-ranking profile* such that status-quo improvement under this profile implies a reduction in the value of the inequality index (henceforth referred to as *inequality reduction*) for the overall economy. This reduction accounts for both teachers who participate in the (re)assignment process and those who do not (Proposition 1).

The natural type-ranking profile is straightforward to construct. First, we partition schools into two sets, $L$ and $H$. Set $L$ comprises schools where marginally assigning high-experience teachers reduces inequality, while set $H$ includes schools where marginally assigning high-experience teachers increases inequality. For schools in $H$, the type ranking prioritizes low-experience teachers over high-experience teachers, whereas for schools in

[7]This index, also known as the quintile share ratio, is one of the primary income inequality measures used by EUROSTAT (European Commission, 2003). Similar ratio indexes are commonly used to capture inequality at the tails of income distributions, as seen in Chancel and Piketty (2021) and Bozio et al. (2024).

$L$, the ranking is reversed.

Conversely, we show that there exist large economies where status-quo improvement under the natural type-ranking profile is necessary for inequality reduction (Proposition 2). Thus, the SI-CC mechanism, when paired with the natural type-ranking profile, always (weakly) reduces inequality in such markets. Our empirical application—the (re)assignment of teachers to schools in France—can be regarded as a large market, as approximately 6.3% of all teachers participated in (re)assignment in our dataset. (see Footnote 4).

Finally, we demonstrate why the SI-CC mechanism utilizing the natural type ranking profile is desirable among all inequality-reducing mechanisms. SI-CC utilizing the natural type ranking profile is a second-best mechanism in the following sense: Any individually rational mechanism that reduces inequality more than SI-CC whenever possible is manipulable by teachers (Proposition 3). Therefore, minimizing inequality is not a goal that can be reached under individually rational and strategy-proof mechanisms (Corollary 2).

**Empirical application: Improving unequal teacher distribution in France.** In the second part of the paper, we quantify the gains that our mechanism can bring by using data on the annual centralized (re)assignment of teachers to the regions of France.[8] This labor market is particularly appropriate to study our question because it suffers from severe imbalance in the distribution of experienced teachers (see the map in Figure A.1).

About 50% of the tenured teachers who ask to change region come from two regions (out of 25) in the suburbs of Paris—called Créteil and Versailles—that are particularly disadvantaged and unattractive for many teachers. To compensate for the large exit flows, most new teachers are assigned to these two regions. This structural imbalance is a serious concern for policymakers. It is frequently raised as a reason for the lack of attractiveness of the teaching profession in France, and it is seen as one of the structural determinants of the large achievement inequalities that France suffers from.[9] Reducing the unequal distribution of teachers across regions has become one of the priorities of French policymakers, as illustrated by this quote from the Ministry that evaluated the teacher transfer process in 2015 (IGAEN, 2015):

> "Inequalities among regions is not considered [by the current (re)assignment process]. When data on mobility is provided, the statistics mostly focus on whether teachers' transfer requests are satisfied, with particular attention being paid to spousal reunion and teachers with a

[8] We consider the (re)assignment of teachers to regions instead of individual schools for reasons that will be clear in the empirical section.

[9] The PISA results show that, in OECD countries, a more socio-economically advantaged student scores 39 points higher in Math than a less-advantaged student, which is equivalent to one year of schooling. There is a large variation between countries in how much a student's social background predicts her school achievement, and France is one of the worst countries in this inequality indicator, ranking fourth from the bottom (OECD, 2012, 2023).

disability. These statistics are necessary, but they do not allow to account for inequalities among regions."

This observation led them to the following recommendation:

"Focus the discussion on mobility around a reduction of inequalities among regions."

**Counterfactuals.** We first empirically estimate preferences separately for tenured and new teachers under a *stability assumption* used by Fack et al. (2019). We then use the estimated teacher preferences, along with data on region priorities and vacant positions, to evaluate the performance of SI-CC. We define a teacher's type as a monotonic function of her experience. To generate the natural type ranking of each region, we start by computing the T20/B20 ratio index using teacher mean experience in each region as the statistic, including the teachers who do not participate in the (re)assignment process. We calculate the ratio of the average teacher experience in the 20% oldest regions over the average experience in the 20% youngest regions.[10] Following our theory, we then identify the sets of $H$ (*older*) and $L$ (*younger*) type regions. The former needs less experienced teachers to reduce inequality. Their natural type ranking orders teachers by decreasing level of experience. The *younger* regions need more experienced teachers to reduce inequality. They rank teachers by increasing level of experience.

As suggested by our theoretical results, the motivation for designing type rankings as above is to ensure that the distribution of teachers becomes more even under SI-CC thanks to the status-quo improvement property. The first empirical outcomes we consider are, therefore, the changes in the inequality index and teacher experience across regions. However, imposing status-quo improvement for regions under the natural type rankings may have a cost in terms of teacher welfare, which motivates us to consider, as additional outcomes, the mobility of tenured teachers and the ranks of the regions that teachers obtain. To quantify the effect of imposing status-quo improvement on the distribution of teachers and on teacher welfare, we compare the allocation under SI-CC to a benchmark mechanism, which is related to SI-CC (and satisfies individual rationality for teachers), but does not satisfy status-quo improvement for regions for a given type ranking profile. This benchmark is a variant of the TTC mechanism of Abdulkadiroğlu and Sönmez (1999). We refer to it as TTC*.

**Empirical performance of SI-CC.** Imposing regional status-quo improvement reduces the value of the inequality index compared to the status quo and it reduces the experience gap between younger and older regions. For example, SI-CC assigns only 1,527 teachers

[10]One of the most widely used measures, the Gini index, has the same sensitivity for inequality between the youngest and oldest regions as between other regions. The Gini index is not a measure of inequality that reflects the Ministry's goals to serve the least advantaged regions. Indeed, the Ministry uses the ratio of high-experience to low-experience teachers in its publications; for example, see our Figure A.1 in the Appendix. Therefore, we use indexes that are more sensitive to inequality in extreme regions, the T20/B20 ratio index in the main analysis, and the maximin index for robustness checks.

with zero or one years of experience to the B20 regions, compared to 2,039 teachers under the benchmark mechanism, TTC*. Consistent with these findings, SI-CC reduces the T20/B20 ratio to 1.3487 from 1.3588 at the status quo, whereas TTC* increases the ratio to 1.3691, thereby exacerbating inequality.

We then examine whether the more equitable distribution of experience achieved by SI-CC comes at the cost of lower teacher welfare. Surprisingly, we find that it does not, at least in terms of the average rank of the regions assigned to teachers. The average rank is lower under SI-CC (9.5$^{\text{th}}$ ranked region) than under TTC* (9.7$^{\text{th}}$ ranked region). However, there are notable differences across teachers. Specifically, while the rank distribution of regions assigned to new teachers under SI-CC Lorenz dominates the distribution under TTC*, the opposite is true for tenured teachers. Thus, this suggests that opting for a more equitable distribution also involves a trade-off between the welfare of new teachers and tenured teachers. However, it also underscores an important advantage of SI-CC as the preference satisfaction of new teachers plays a crucial role in making the teaching profession more attractive (Cour des Comptes, 2013, 2017).

Overall, our results show that SI-CC significantly enhances the distribution of teachers, boosting the welfare of new teachers while reducing that of tenured teachers. This provides empirical support for our new approach, where school priorities and the assignment mechanism serve as tools to reduce inequality in teacher distribution, in contexts where SI-CC is the implemented mechanism.

**Increasing teacher mobility under SI-CC.** Our empirical analysis identifies an important trade-off between tenured teacher mobility (in particular from disadvantaged regions) and inequality reduction. However, we show next that, if the policymaker wants to keep tenured teacher mobility high, this can be achieved at the expense of a slightly lower inequality reduction. To do so, we develop a modified version of SI-CC, called SI-CC*, in which new teachers have to rank the youngest regions at the top of their preferences (and otherwise preserving their submitted ranking within young and old regions). As we discuss later, the current mechanism used in France implicitly causes new teachers to go down their preference lists until they apply to the youngest regions. By requiring them to rank these regions first, SI-CC* aligns with the current policy approach toward new teachers. In Proposition A.5 (see Appendix F), we show that this mechanism is strategy-proof as it keeps the reported relative ranking within each group intact, and the definition of the youngest regions is exogenously fixed. We also show it is status-quo improving for regions, and, hence, inequality reducing under the natural type rankings in large markets. And last, it is individually rational when new teachers are assumed to find all regions acceptable.[11]

[11] The French Ministry of Education completes new teachers' lists to ensure all regions are ranked and vacancies filled. Given negligible dropout rates even though these teachers are assigned to the least attractive

In the final counterfactual analysis, we compare SI-CC* with SI-CC and the Current French mechanism. The latter is based on the teacher-proposing deferred acceptance algorithm of Gale and Shapley (1962), in which the region priorities use the Ministry's point scheme (primarily increasing based on teacher experience level, see Appendix F.2.1). Moreover, it modifies these priorities to ensure the mechanism's individual rationality for tenured teachers.

Our results show that, while the inter-region mobility of tenured teachers under SI-CC* is closer to the mobility under the Current French mechanism (1,307 vs 1,711, respectively, whereas only 986 tenured teachers move under SI-CC), SI-CC* achieves inequality reduction at levels comparable to SI-CC (2.75% vs 2.79% decreases of the initial 35.88% difference in the experience ratio, respectively). In contrast, the Current French mechanism increases inequality relative to the status quo, corresponding to a 6.72% increase in the initial 35.88% difference in the T20/B20 experience ratio.

# 2 Model

Let $T$ be a finite set of **teachers** and $S$ be a finite set of **schools**. Each teacher is seeking employment or re-employment at one of the schools, which are managed by a policy-maker. Each teacher $t$ has a **type** which captures her observable characteristics, such as experience, education, past performance, etc., or only a subset of these. Let $\Theta$ be the finite type space and $\tau : T \to \Theta$ be the **type function** such that $\tau(t)$ is the type of teacher $t$. For any $\hat{T} \subseteq T$, we define $\hat{T}^{\theta} := \{t \in \hat{T} \,:\, \tau(t) = \theta\}$.

In addition to $\Theta$, we define the **vacant seat type**, denoted by $\theta_{\emptyset}$, as a special type which will be used for vacant seats of a school. Each school $s$ has a **capacity** of $q_s$. Let $q = (q_s)_{s \in S}$. Each teacher $t$ has a **strict preference relation**, which is a linear order and denoted by $P_t$, over the schools and **outside option** denoted by $\emptyset$. Let $P = (P_t)_{t \in T}$. We denote the at least as good as relation related to $P_t$ by $R_t$ for every teacher $t$: $s \, R_t \, s'$ if and only if $s = s'$ or $s \, P_t \, s'$.

A **matching** $\mu : T \to S \cup \{\emptyset\}$ is a function such that $|\mu^{-1}(s)| \leq q_s$. We will occasionally use a set-based definition of functions to denote matchings: $\mu := \big\{ \big(t, \mu(t)\big) : t \in T \text{ s.t. } \mu(t) \neq \emptyset \big\}$. Let $\mathcal{M}$ be the set of matchings. With a slight abuse of notation, we use $\mu_t$ and $\mu_s$ instead of $\mu(t)$ and $\mu^{-1}(s)$, respectively. Thus, $\mu_s^{\theta}$ is the set of teachers of type $\theta \in \Theta$ that are assigned school $s$. For the vacant seat type $\theta_{\emptyset}$, we define $\mu_s^{\theta_{\emptyset}}$ as the set of vacant seats under matching $\mu$ at school $s$, and $|\mu_s^{\theta_{\emptyset}}| := q_s - |\mu_s|$.[12] We refer to $\mu_t$ as the **assignment** of teacher $t$ and $\mu_s$ as the **assignment** of school $s$ in matching $\mu$. For a subset of teachers $\hat{T}$, we denote the set of their matches in $\mu$ by $\mu_{\hat{T}}$.

regions, it seems reasonable to assume all schools are acceptable to new teachers.

[12]The identity of vacant seats does not matter for our purposes, and we will mainly use the cardinality of this set in our model.

Initially, some teachers are already employed by some schools. This is captured by a **status-quo matching** $\omega$. If $\omega_t = s$, then teacher $t$ is a **status-quo employee** of school $s$. If $\omega_t = \emptyset$, then teacher $t$ is called a **new teacher**, who is unemployed at the status quo. By definition, $|\omega_s| \leq q_s$ for each school $s$. We denote the set of new teachers by $N$.

Teachers who are status-quo employees of schools are referred to as **tenured teachers**. We make one assumption on teacher preferences: we assume that each tenured teacher finds her status-quo assignment acceptable, i.e., $\omega_t \, P_t \, \emptyset$ for each $t \in T \setminus N$.

We refer to the list $\langle T, \Theta, \tau, S, q, \omega, P \rangle$ as a **teacher (re)assignment market**. Only teacher preferences are private information in our application. Thus, when there is no ambiguity, we denote a market by teacher preferences $P$. We are seeking a matching given a market $P$. We next introduce desirable properties of matchings.

A matching $\mu$ is **individually rational** if every teacher weakly prefers her assignment under $\mu$ to her assignment under $\omega$, i.e., $\mu_t \, R_t \, \omega_t$ for every $t \in T$.

The policymaker endows each school $s$ with a **type ranking**, which is a reflexive, complete, and transitive binary order and denoted by $\trianglerighteq_s$, over the types of teachers and the vacant seat type. We allow type rankings to induce indifference between types, including the vacant seat type. Let $\sim_s$ and $\triangleright_s$ denote the symmetric and asymmetric parts of $\trianglerighteq_s$. In particular, if $\theta \sim_s \theta'$, then $\theta \trianglerighteq_s \theta'$ and $\theta' \trianglerighteq_s \theta$. If $\theta \triangleright_s \theta'$, then $\theta \triangleright_s \theta'$ but not $\theta' \triangleright_s \theta$. Let $\trianglerighteq = (\trianglerighteq_s)_{s \in S}$ be the type ranking profile.

Given two matchings $\mu$ and $\gamma$, we say $\mu$ **Lorenz dominates** $\gamma$ **under** $\trianglerighteq_s$ if for each type $\theta \in \Theta \cup \{\theta_\emptyset\}$, $\sum_{\theta' \trianglerighteq_s \theta} |\mu_s^{\theta'}| \geq \sum_{\theta' \trianglerighteq_s \theta} |\gamma_s^{\theta'}|$. We say that $\mu$ is an **unambiguous weak improvement over** $\gamma$ **under** $\trianglerighteq_s$ for school $s$ if (1) $|\mu_s| \geq |\gamma_s|$ and (2) $\mu_s$ Lorenz dominates $\gamma_s$ under $\trianglerighteq_s$. A matching $\mu$ is **status-quo improving for schools under** $\trianglerighteq$ if $\mu$ is an unambiguous weak improvement over $\omega$ under $\trianglerighteq_s$ for every school $s \in S$.[13]

A matching $\mu$ is **status-quo improving under** $\trianglerighteq$ if it is status-quo improving for schools under $\trianglerighteq$ and individually rational. We denote the set of status-quo improving matchings under $\trianglerighteq$ by $\mathcal{M}^{SI}(\trianglerighteq)$.

An important reason why we focus on status-quo improvement for schools as a desirable property is that when used in conjunction with a carefully designed type ranking

[13]If we used a more stringent binary relation than Lorenz domination in the definition of status-quo improvement, such as using a responsive order, then as we demonstrate in Appendix E through an example, we obtain incompatibility between our desired properties. More importantly, in Section 4, we demonstrate that Lorenz domination is crucial to establishing a link between inequality reduction and status-quo improvement. On the other hand, Condition (1) in the definition of unambiguous weak improvement can be dispensed of. All our analysis would go through with an appropriate change to the mechanism we introduce in the next section. Moreover, our two versions of the mechanism, respecting unambiguous weak improvements with and without Condition (1), led to similar empirical outcomes, in a market with overdemand from teachers that have better types than vacant seats, as in the French market analyzed in our empirical section. Since not allowing a school to have fewer teachers than it had at the status quo is more fitting with the status-quo improvement idea, we keep Condition (1).

profile for a given inequality measure, it is both sufficient and necessary in large markets to achieve inequality reduction, a central objective of the policymaker in our main application. We carry out a formal analysis to this end in Section 4.

Next, we define our efficiency notion. Given two matchings $\mu$ and $\gamma$, $\mu$ **Pareto dominates** $\gamma$ if $\mu_t \, R_t \, \gamma_t$ for all $t \in T$, and $\mu_{t'} \, P_{t'} \, \gamma_{t'}$ for some $t' \in T$. A matching is **Pareto efficient** if it is not Pareto dominated by any other matching. Let $\overline{\mathcal{M}}$ be a subset of matchings. A matching $\mu$ is **constrained efficient in** $\overline{\mathcal{M}}$ if $\mu \in \overline{\mathcal{M}}$ and it is not Pareto dominated by any other matching in $\overline{\mathcal{M}}$. Thus, given a type ranking profile $\trianglerighteq$, a matching is **SI-constrained efficient under** $\trianglerighteq$ if it is constrained efficient in $\mathcal{M}^{SI}(\trianglerighteq)$, i.e., there is no other status-quo improving matching under $\trianglerighteq$ that Pareto dominates it.

We inspect rules that select a matching for each market. Formally, a (**direct**) **mechanism** $\varphi$ is a function that chooses a matching for any market $P$. Let $\varphi(P)$, $\varphi_t(P)$, and $\varphi_s(P)$ denote the matching selected by mechanism $\varphi$ in market $P$, the assignment of teacher $t$, and the assignment of school $s$ in that matching, respectively.

A mechanism $\varphi$ is **strategy-proof** if truth-telling is a weakly dominant strategy for all teachers, that is, for each market $P$, teacher $t$, and preference report $P'_t$, $\varphi_t(P_t, P_{-t}) \; R_t \; \varphi_t(P'_t, P_{-t})$, where $P_{-t} = (P_{t'})_{t' \in T \setminus \{t\}}$.

# 3 An SI-Constrained Efficient Mechanism

In this section, we introduce an SI-constrained efficient and strategy-proof mechanism. To achieve this goal, we introduce additional tools.

Our mechanism will iteratively construct a sequence of directed graphs in which teachers, schools, and the outside option are the nodes. Teachers can only point to schools or the outside option and schools can only point to their status-quo employees in each of these graphs. Our mechanism relies on *executing* two types of multi-lateral exchanges based on the constructed directed graphs.

A **cycle** is a directed path of distinct teachers $\{t_\ell\}_{\ell \in \{1,\ldots,k\}}$ and options $\{x_\ell\}_{\ell \in \{1,\ldots,k\}} \subseteq S \cup \{\emptyset\}$, denoted as $(x_1, t_1, x_2, t_2, \ldots, x_k, t_k)$ such that for all $\ell$, if $x_\ell$ is a school, $\omega_{t_\ell} = x_\ell$, each node points to the next node in the path, and $t_k$ points back to $x_1$.

A **chain** is a directed path of distinct teachers $\{t_\ell\}_{\ell \in \{0,\ldots,k-1\}}$ and (not necessarily distinct) schools $\{s_\ell\}_{\ell \in \{1,\ldots,k\}}$, denoted as $(t_0, s_1, t_1, \ldots, s_{k-1}, t_{k-1}, s_k)$ such that for all $\ell < k$, $\omega_{t_\ell} = s_\ell$ and each node points to the next node in the path. Here, the chain starts with $t_0$ and ends with $s_k$.

As certain cycles and chains are encountered in the constructed graph, we will **execute** the exchanges in them by assigning each teacher to the school or the outside option to which she is pointing and remove her.

We will construct a sequence of matchings $\mu^0 := \emptyset \subsetneq \mu^1 \subsetneq \mu^2 \subsetneq \ldots \subsetneq \mu^K$ through the iterative algorithm that calculates the outcome of our mechanism. In each Step $k$, as teachers are assigned in $\mu^k$, they will be removed; similarly, schools whose all positions are filled in $\mu^k$ will be removed. Let $T^k$ and $S^k$ be the sets of remaining teachers and schools at the beginning of Step $k$, respectively. We initialize them as $T^1 := T$ and $S^1 := S$.

For each Step $k \geq 1$, school $s$, and type $\theta \in \Theta$, let $b_s^{k,\theta}$ track the **balance** of number of type $\theta$ teachers at school $s$ in $\mu^{k-1}$. This balance is defined as the difference between the number of type $\theta$ teachers in $\mu_s^{k-1}$ and in $\omega_s$, who got assigned in $\mu^{k-1}$:

$$b_s^{k,\theta} := \left| \; [\mu_s^{k-1}]^\theta \; \right| - \left| \; [\omega_s \setminus T^k]^\theta \right|.$$

Similarly, we define the balance of vacant seat type, $\theta_\emptyset$, at school $s$ in $\mu^{k-1}$ as follows:

$$b_s^{k,\theta_\emptyset} := -\Big( \underbrace{(q_s - |\omega_s|)}_{=|\omega_s^{\theta_\emptyset}|= \text{\# initial vacant seats}} - \underbrace{(q_s - |\mu_s^{k-1}| - |\omega_s \cap T^k|)}_{\text{net \# current vacant seats}} \Big) = -\big( \underbrace{|\mu_s^{k-1}| - |\omega_s \setminus T^k|}_{\text{net \# currently filled vacant seats}} \big).$$

Therefore, $b_s^{0,\theta} = 0$ for all $\theta \in \Theta \cup \{\theta_\emptyset\}$. Let $b^k = (b_s^{k,\theta})_{\theta \in \Theta \cup \{\theta_\emptyset\}, s \in S}$.

One of our main theoretical innovations relies on designing *pointing rules* that designate which possible directed edges will form in the algorithm.

Fix a type ranking profile $\trianglerighteq$. The pointing rule of schools relies on their type rankings and a given tie breaker. Formally, a **tiebreaker** is a linear order $\vdash$ over teachers.[14] It can be randomly determined or can be the mandated priority orders for a particular application, such as in the French case, or can be exogenously fixed in some other manner. For each school $s$, using tiebreaker $\vdash$ and its type ranking $\trianglerighteq_s$, we first construct a **pointing order** $\gtrdot_s$ over teachers in $\omega_s$, which is a linear order: For any two distinct teachers $t, t' \in \omega_s$,

$$t \gtrdot_s t' \iff \tau(t) \lhd_s \tau(t') \text{ or } \left[\tau(t) \sim_s \tau(t') \text{ and } t \vdash t'\right].$$

Note that a worse-type teacher is prioritized over a better-type teacher under $\gtrdot_s$, and only when teachers with types for which $s$ is indifferent are compared, we use the tiebreaker to prioritize one over the other.

As the mechanism will iteratively assign and remove teachers, the **pointing rule of schools** is "point to the highest remaining priority teacher in its pointing order."

In Step $k$, any teacher $t \in T^k$ is allowed to point to a school $s$ as long as her assignment to $s$, possibly replacing the teacher pointed to by $s$, would not violate the status-quo improvement requirement for $s$ when we consider the teachers in $\mu_s^{k-1}$ and $\omega_s \cap T^k$. In particular, let $A_t^k$ be the opportunity set for $t$, that is, the set of schools to which $t$ can point in this step together with the outside option $\emptyset$. A school $s \in S^k$ is included in $A_t^k$ if (and only if) at least one of the following two **school improvement conditions** holds:

1. (Improvement for $s$ by teacher trades) if $s$ points to some teacher $t^*$, and either $\tau(t) \trianglerighteq_s$

[14]Technically, each tiebreaker induces a new mechanism in our class.

$\tau(t^*)$ or

$$\sum_{\theta' \trianglerighteq_s \theta} b_s^{k,\theta'} > 0 \text{ for all types } \theta \in \Theta \cup \{\theta_\emptyset\} \text{ such that } \tau(t^*) \trianglerighteq_s \theta \vartriangleright_s \tau(t),$$

or

2. (Improvement for $s$ by only incoming teachers) school $s$ currently has a vacant position, i.e., $q_s - |\mu_s^{k-1}| > |\omega_s \cap T^k|$, there are remaining new teachers, and either $\tau(t) \trianglerighteq_s \theta_\emptyset$ or

$$\sum_{\theta' \trianglerighteq_s \theta} b_s^{k,\theta'} > 0 \text{ for all types } \theta \in \Theta \cup \{\theta_\emptyset\} \text{ such that } \theta_\emptyset \trianglerighteq_s \theta \vartriangleright_s \tau(t).$$

When the above first (or second) condition holds, we refer to it as "Improvement Condition 1 (or 2) holds for $s$ via $t$."

Now, we are ready to define our mechanism through an iterative algorithm:

**Definition 1.** ***Status-quo Improving Cycles and Chains (SI-CC) Mechanism Induced by*** $\trianglerighteq$ ***and*** $\vdash$***:*** *Determine the pointing order $\gg_s$ for each school $s$ using $\trianglerighteq_s$ and $\vdash$.*

***Step*** $k \geq 1$***:***

- *Each $s \in S^k$ points to the highest priority teacher in $\omega_s \cap T^k$ under $\gg_s$, if $\omega_s \cap T^k \neq \emptyset$, denoted by $t^s := \max_{\gg_s} \omega_s \cap T^k$. Otherwise, $s$ does not point to any teacher.*
- *Each $t \in T^k$ points to her most preferred option in $A_t^k$, denoted by $x^t := \max_{P_t} A_t^k$.*
- *Outside option $\emptyset$ points to all teachers who are pointing to it.*

*There are two possible cases:*

***Case (i).*** *There is a cycle in which for each school in the cycle Improvement Condition 1 holds via the pointing teacher to it or a cycle between a single teacher and the outside option $\emptyset$:*
*Each teacher can be in at most one such cycle as she points at most to a single option. We execute exchanges in each such cycle encountered by assigning each teacher $t$ in that cycle to $x^t$, update $\mu^{k-1}$ by including the new assignments to form $\mu^k$, remove assigned teachers in this step from $T^k$ and filled schools in this step from $S^k$ to form $T^{k+1}$ and $S^{k+1}$, respectively, and go to Step $k+1$.*

***Case (ii).*** *There exists no such cycle:*
*There must be a chain (built in the following lines) and $N \cap T^k \neq \emptyset$.*[15] *We select the chain $(t_0, s_1, t_1, \ldots, s_{\ell^*-1}, t_{\ell^*-1}, s_{\ell^*})$ for some $\ell^*$ determined by the steps below:*

- *Chain construction step 0: Select as the first teacher of the chain the new teacher $t_0 = \max_\vdash N \cap T^k$, i.e., the highest priority remaining new teacher under the tiebreaker, and*

[15]Since Case (i) does not hold, no teacher points to the outside option. If via each teacher Improvement Condition 1 holds for the school she is pointing to, there is a cycle in which, for all schools in the cycle, Improvement Condition 1 holds, which contradicts the definition of Case (ii). Therefore, at least via one teacher Improvement Condition 2 holds for the school she is pointing to. Then, by definition of this condition, there must remain a new teacher.

*then include school $x^{t_0}$ as $s_1 = x^{t_0}$ in the chain.*[16] *If Improvement Condition 1 does not hold for $s_1$ via $t_0$, but only Improvement Condition 2 holds, then we end the chain with $\ell^* = 1$; otherwise, we continue with the chain construction step 1.*

– *Chain construction step $\ell$ (for $\ell \geq 1$): Include to the chain teacher $t^{s_\ell}$, i.e., the teacher pointed to by the last included school $s_\ell$, as $t_\ell = t^{s_\ell}$*[17] *and school $x^{t_\ell}$, i.e., the school pointed to by $t_\ell$, as $s_{\ell+1} = x^{t_\ell}$. If Improvement Condition 1 does not hold for $s_{\ell+1}$ via $t_\ell$, but only Improvement Condition 2 holds, we terminate the construction with $\ell^* = \ell + 1$.*[18] *Otherwise, we continue with the chain construction step $\ell + 1$.*

*The last included school $s_{\ell^*}$ terminates the selected chain.*

*We execute the exchanges in the selected chain by assigning each teacher in the chain to the school to which she is pointing, update $\mu^{k-1}$ by including the new assignments to form $\mu^k$, remove assigned teachers in this step from $T^k$ and filled schools in this step from $S^k$ to form $T^{k+1}$ and $S^{k+1}$, respectively, and go to Step $k+1$.*

*The mechanism terminates when all teachers are removed, say in Step K. Its outcome is the final matching $\mu^K$.*

The name of the mechanism suggests that individual rationality for teachers and status-quo improvement for schools are simultaneously satisfied. Indeed, this is the case. We introduced several innovations in the mechanism that exploit improvement possibilities over the status-quo matching for both teachers and schools.

Individual rationality is straightforward to show. A tenured teacher only points to a school at least as good as her status-quo employer: as for this school Improvement Condition 1 always holds via her, she will, at worst, form a cycle of size two with this school when it points to her. A new teacher never points to a school to which she prefers remaining unmatched. Additionally, observe that each teacher is assigned the best option to which she can point in the step she is assigned.

What is more delicate is the status-quo improvement of schools, that is, how we make sure that they always weakly improve with respect to their status-quo assignment in every step. This is ensured through both teacher and school pointing rules.

A school's pointing order designates in which order the school would like to *send out* its status-quo employees. By pointing, the school effectively gives permission to one of its status-quo employees to be assigned possibly to a different school. Thus, we ensure this priority order respects the reverse of its type ranking: Lower ranked-type employees are pointed to first, and higher ranked-type employees are pointed to later. This is the first

[16] Such a school exists, because if she does not point to a school, then she pointed to the outside option $\emptyset$ which contradicts the definition of Case (ii).

[17] Such a teacher exists by Improvement Condition 1.

[18] This iterative procedure is guaranteed to terminate. Otherwise, we would have a cycle in which for each school in the cycle Improvement Condition 1 holds via the teacher pointing to it, a contradiction as Case (i) does not hold.

innovation.

The teacher pointing rule designates which teachers can be assigned to a school. A teacher can point to a school only if she can unambiguously weakly improve the school's assignment over its status quo by replacing the employee pointed to by the school (Condition 1) or occupying a vacant seat (Condition 2).

Under Condition 1, if the type of the pointing teacher is at least as good as the type of the pointed employee, a school has no danger of becoming worse off if this replacement goes through. However, we sometimes allow the type of the pointing teacher to be worse than that of the pointed one. Such a teacher is allowed to point when the following holds. As trades improve a school over its status quo during the execution of the algorithm, schools may acquire teachers who are of better types than the outgoing status-quo employees. Therefore, they may build up a *buffer* so that replacing the employee it is currently pointing to with a worse-type teacher can still status-quo improve the school, although this replacement makes it worse off with respect to the previous step. The existence of the buffer is tracked by checking whether the sums of the relevant type balances are strictly positive through Condition 1. The use of this buffer and allowing the schools sometimes to be pointed to by a worse-type teacher than their currently pointed employee help to achieve constrained efficiency.

While the first condition is about a trade the school will make by exchanging an outgoing teacher with an incoming teacher, Condition 2 is only relevant as long as new teachers remain in the algorithm. When Condition 2 holds for a school via some teacher, but not Condition 1, the school will not send out an employee as it has extra capacity: it will only hire one additional teacher by guaranteeing status-quo improvement.

We illustrate how the algorithm of the SI-CC mechanism works in Example A.1 in Appendix D. We are ready to state our main result in this section.

**Theorem 1.** *For any type ranking profile $\trianglerighteq$ and tiebreaker $\vdash$, the SI-CC mechanism is strategy-proof and SI-constrained efficient under $\trianglerighteq$.*

SI-constrained efficiency of the mechanism is delicate to show. Note that SI-constrained efficiency implies that the mechanism outcome is Pareto undominated for teachers among all status-quo improving matchings. However, the pointing rule for teachers has restrictions imposed by the school improvement conditions. That is, a teacher cannot arbitrarily point to the best school she likes. We show that the restrictions imposed by these conditions are the necessary and sufficient conditions to maintain status-quo improvement for schools without causing the outcome to be Pareto dominated for teachers. Therefore, implementing any further Pareto improvement for teachers would make the schools worse off with respect to the status quo. Moreover, imposing further restrictions for teacher pointing would prevent SI-constrained efficiency.

The strategy-proofness of the mechanism relies on several observations. First, once a teacher is pointed to by a school, she will continue to be pointed to until she is assigned. We show that the opportunity set for each teacher $t$, $A_t^k$, weakly shrinks through steps $k = 1, \ldots$. Although Improvement Conditions 1 or 2 can stop holding for a school via a teacher $t$ through steps, we show that by submitting different preferences, teacher $t$ cannot affect which schools leave and stay in $A_t^k$ before she is assigned.

According to the pointing rule of schools, in each step of SI-CC, each school points to one of its employees who has the lowest-ranked type. One can wonder whether Theorem 1 holds when we consider alternative school pointing rules under the SI-CC mechanism. Example A.2 in Appendix D shows that under any alternative pointing rule, a teacher can manipulate SI-CC and it is no longer SI-constrained efficient.

# 4 Type Ranking Design to Reduce Inequality

As discussed in the Introduction, one of the main objectives of the policymaker is to reduce inequalities across schools, for instance, in terms of the distribution of experienced teachers (measured through their types). In this section, we give a foundation to our concept of status-quo improvement by showing that it is directly related to inequality reduction.

## 4.1 Inequality Measures for Teacher (Re)assignment

We start by introducing inequality measures to our setup. Before proceeding, it is important to enrich the framework presented thus far. This is necessary to account for an important aspect of the teacher assignment application: a significant number of employees in each school do not apply for reassignment. In practice, these non-participants constitute the vast majority of teachers and must be considered when measuring inequality across schools. These teachers can be viewed as previously assigned their most preferred school and, as a result, have no incentives to engage in the (re)assignment market.[19] To capture this feature, for each school we define the sets of teachers employed by the school who are non-participants in the (re)assignment process. Given a set of schools $S$ and types $\Theta$, we define the profile of **non-participating employees** $E = (E_s)_{s \in S}$ where for each school $s$, we have $E_s = (E_s^\theta)_{\theta \in \Theta}$ so that $E_s^\theta$ is the set of non-participating employees of school $s$ with type $\theta$. With a slight abuse of notation, we also use $E_s$ to refer to the set of all non-

[19]When explicitly modeling non-participating teachers, one may consider endogenous participation—teachers may choose to participate based on their prospects in the (re)assignment market. Participation (followed by truth-telling) is a dominant strategy under mechanisms that are individually rational and strategy-proof. Non-participation and participation (followed by truth-telling) yield the same outcome for teachers already at their most preferred school. Assuming that (1) teachers will follow their dominant strategy to participate if they are not currently at their most preferred school, and (2) those already assigned to their preferred school will opt out of participation, decisions to participate are insensitive to changes in the assignment mechanism. See Section 7 for further discussion in the light of empirical findings.

participating employees of $s$, i.e., $E_s := \bigcup_{\theta \in \Theta} E_s^\theta$.[20] As before, let $T$ be the set of teachers **participating** in the (re)assignment process.

Let $K := |\Theta|$ and let $\Theta := \{\theta_0, \theta_1, \ldots, \theta_{K-1}\}$. Each type $\theta \in \Theta$ is associated to a **type value** $v_\theta \in \mathbb{R}_+$ which is a non-negative real number. We let $v_k := v_{\theta_k}$ and we label types and values so that $v_0 := 0 < v_1 < v_2 < \cdots < v_{K-1}$. We also let $\mathcal{V} := \{v_0, \ldots, v_{K-1}\}$. These values will be used to compare assignments between schools. As an example, one can think of a type value as the teachers' years of experience (which may be bundled together as we do in Section 5.3 in our empirical simulations). The vacant seat type $\theta_\emptyset$ has a value of zero so that $v_\emptyset := v_0 = 0$. To make sense of this assumption, let us note that the teachers with lowest experience type $\theta_0$ are new teachers. Further, in practice, if a position is left vacant by the mechanism, it is filled with a substitute teacher. Our assumption is that new teachers with the lowest experience and substitute teachers are similar.[21]

Consider a (re)assignment market $\langle T, \Theta, \tau, S, q, \omega, P \rangle$. Including the profile of non-participating teachers $E$ and the value profile over types, we refer to $\langle T, \Theta, \tau, S, q, \omega, P; E, \mathcal{V} \rangle$ as an **economy**. We say that the (re)assignment market is **associated** with the economy. As before, when we refer to matchings (including status-quo matching $\omega$), we assume that they are defined for teachers in $T$, i.e., for participants. Thus, at any matching $\mu \in \mathcal{M}$, $\mu_s \cup E_s$ is the final set of teachers employed by school $s$.

Fix a school $s$. A **value distribution for school** $s$ is a vector $\delta_s = (\delta_s^v)_{v \in \mathcal{V}} \in \Delta^{K-1}$, the simplex of dimension $K-1$, i.e., $\delta_s^v \in [0,1]$ for each $v \in \mathcal{V}$ and $\sum_{v \in \mathcal{V}} \delta_s^v = 1$. Each matching $\mu$ induces a value distribution for school $s$ denoted by $\delta_s^\mu = (\delta_s^{\mu,v})_{v \in \mathcal{V}} \in \Delta^{K-1}$ such that for each $v_k \in \mathcal{V}$,

$$\delta_s^{\mu, v_k} := \begin{cases} \frac{|E_s^{\theta_k}| + |\mu_s^{\theta_k}|}{|E_s| + q_s} & \text{if } k > 0 \\ \\ \frac{|E_s^{\theta_k}| + |\mu_s^{\theta_k}| + (q_s - |\mu_s|)}{|E_s| + q_s} & \text{if } k = 0 \end{cases}.$$

For $k > 0$, $\delta_s^{\mu, v_k}$ refers to the fraction of teachers (participants and non-participants) with type value $v_k$ assigned to $s$ under $\mu$. For $k = 0$, recall that $|\mu_s^{\theta_\emptyset}| = q_s - |\mu_s|$ is the number of vacant positions and $\delta_s^{\mu,0}$ is the fraction of positions either vacant or occupied by a teacher with a type value 0.[22]

We give the definition of Lorenz domination for value distributions here, which is equivalent to first-order stochastic dominance if the value distribution were a probability

[20] Naturally, we assume that $E_s^{\theta_\emptyset} = \emptyset$.

[21] As presented in Section 5.3, 75.18% of new teachers have zero years of experience. Hence, 24.82% of new teachers have a positive experience, sometimes higher than tenured teachers. For example, these are teachers migrating from private or international school systems or who have taken a leave from the profession and are coming back.

[22] While not needed for our results, note that we assumed that $v_0 = 0$.

measure. For any two value distributions $\delta_s$ and $\hat{\delta}_s$, we say that $\delta_s$ **Lorenz dominates (LDs)** $\hat{\delta}_s$ if $\sum_{\ell=0}^{k} \delta_s^{v_{K-1-\ell}} \geq \sum_{\ell=0}^{k} \hat{\delta}_s^{v_{K-1-\ell}}$ for all $k \in \{0, \ldots, K-1\}$. We say that $\delta_s$ **strictly Lorenz dominates (strictly LDs)** $\hat{\delta}_s$ if $\delta_s$ LDs $\hat{\delta}_s$ and the inequality above is strict for some $k$.

To compare inequality across schools, we use summary statistics to quantify each school's value distribution. A **statistic** is a function $f : \Delta^{K-1} \to \mathbb{R}$.[23] A statistic $f$ is **Lorenz-dominance increasing (LD-increasing)** if, for any two value distributions $\delta_s$ and $\hat{\delta}_s$, we have $\delta_s$ LDs $\hat{\delta}_s$ implies $f(\delta_s) \geq f(\hat{\delta}_s)$. We say that $f$ is **strictly LD-increasing** if, for any two value distributions $\delta_s$ and $\hat{\delta}_s$, we have $\delta_s$ strictly LDs $\hat{\delta}_s$ implies $f(\delta_s) > f(\hat{\delta}_s)$. Clearly, the mean statistic defined by $f(\delta_s) = \sum_{k=0}^{K-1} v_k \, \delta_s^{v_k}$ is LD-increasing. Many other statistics, including the median statistic, satisfy this condition.[24]

Across all schools, let $\delta = (\delta_s)_{s \in S} \in \left(\Delta^{K-1}\right)^{|S|}$ denote a profile of value distributions. Given a statistic $f$, we denote its vector of outcomes for $\delta$, with a slight abuse of notation, as $f(\delta) := \left(f(\delta_s)\right)_{s \in S}$.

A statistic $f$ is **continuous** if for any sequence of profiles of value distributions $\{\delta^n\}$ such that $\delta^n \to \delta^*$ for some $\delta^* \in \left(\Delta^{K-1}\right)^{|S|}$, we have that $f(\delta^n) \to f(\delta^*)$.[25] For example, both the mean and median statistics are continuous.

With the notion of a statistic in our hands, we can define our inequality measure. An **inequality index** is a function $\mathcal{I} : \mathbb{R}^{|S|} \to \mathbb{R}$.[26] Here, a point in the domain $z = (z_s)_{s \in S} \in \mathbb{R}^{|S|}$, is typically the value of a statistic $f$ evaluated as a vector function at some matching $\mu$ for all schools, i.e., $z = f(\delta^\mu)$ where $\delta^\mu = (\delta_s^\mu)_{s \in S}$. Naturally, we assume the policymaker prefers a lower value of an inequality index to a higher one. We assume that inequality indices are continuously differentiable on an open and dense subset of $\mathbb{R}^{|S|}$. A well-known example of an inequality index is the weighted Gini index (c.f. Gini, 1912 and Bhattacharya and Mahalanobis, 1967; also see Sen, 1973) defined as $\mathcal{G}(z) := \frac{1}{2\sum_{s \in S} w_s \, z_s} \sum_{s \in S} \sum_{s' \in S} |z_s - z_{s'}| \, w_s \, w_{s'}$ where $w_s$ is the weight associated to school $s$ such as the proportion of teachers that can be employed by the schools, $w_s := \frac{|E_s| + q_s}{\sum_{s' \in S} |E_{s'}| + q_{s'}}$.[27] It is often argued that the Gini index is not sensitive enough to the "tail" of the distribution (see Atkinson et al., 2011). Two other indices used in practice to overcome this issue are

[23]Strictly speaking, a statistic should take the number of types $K$ as input. For notational convenience, we ignore this dependence.

[24]For a value distribution $\delta_s$, the median statistic $f$ is defined as the real value $x$ satisfying $\sum_{v \leq x} \delta_s^v \geq \frac{1}{2}$ and $\sum_{v \geq x} \delta_s^v \geq \frac{1}{2}$. When there are multiple such values, we let $x$ be the mean over all possible such values.

[25]Recall that each profile of value distributions $\delta$ is a member of $\left(\Delta^{K-1}\right)^{|S|}$. We simply endow $\Delta^{K-1}$ with the standard topology of weak convergence of measures. The positive real line is endowed with the standard Euclidean topology, while any product set is endowed with the product topology.

[26]Strictly speaking, an inequality index should take the number of schools $|S|$. For notational convenience, we implicitly use this dependence.

[27]Note that the Gini index is continuously differentiable on an open and dense set of points even though it may be non-differentiable at some points.

as follows. The T$X$/B$Y$ ratio index (where $X \in (0, 100)$, $Y \in (0, 100 - X]$), which takes the ratio between the average of the statistics values above the top $X^{\text{th}}$ percentile over the average of the statistics values below the bottom $Y^{\text{th}}$ percentile (i.e., top $100 - Y^{\text{th}}$ percentile),[28] and the maximin index (also known as the Atkinson-$\infty$ index, Atkinson, 1970)[29] defined as $1 - \frac{\min_{s \in S} z_s}{\sum_{s \in S} w_s z_s}$.

## 4.2 A Foundation for Status-quo Improvement in Large Markets: SI-CC as a Second-Best Mechanism

Our goal in this section is to show that for a natural type ranking profile $\trianglerighteq$, status-quo improvement for schools under $\trianglerighteq$ is both necessary and sufficient in some generic sense for inequality reduction measured by a standard inequality index (such as the Gini index) based on a standard welfare statistic (such as mean experience at a school) in large economies.

We consider an economy $\langle T, \Theta, \tau, S, q, \omega, P; E, \mathcal{V} \rangle$ and allow the profile of non-participating employees to vary. More specifically, we consider a sequence of non-participating employee profiles $\{E^n\}_{n \geq 1}$ such that for each $n \geq 1$, the economy $\langle T, \Theta, \tau, S, q, \omega, P; E^n, \mathcal{V} \rangle$ is called an $n$**-economy** and is simply denoted as $E^n$ such that for each school $s$ and type $\theta$, we have $\left|E_s^{\theta,n}\right| := n \left|E_s^{\theta}\right|$ where $E^1 := E$. We refer to $\langle T, \Theta, \tau, S, q, P, \omega; E^1, \mathcal{V} \rangle$ as the **base economy**. Therefore, an $n$-economy has $n$ replicas of each non-participating employee of each school at the base economy. Given a matching $\mu$ of the associated (re)assignment market, for each school $s$, assuming that $E_s^{\theta} \neq \emptyset$ whenever $\mu_s^{\theta} \neq \emptyset$ for any type $\theta$, the economy without participants becomes "dominant", i.e., $\frac{\left|E_s^{\theta,n}\right|}{\left|\mu_s^{\theta}\right|}$ goes to $\infty$ as $n$ increases.[30] We note that the set of non-participants is often quite large in applications, e.g., they represent about 93.7% of the whole set of teachers in our application on the assignment of French teachers,[31] and so assuming a dominant non-participant market (i.e., $n$ is large, as we will do) is certainly a good approximation in

[28] Formally, fix the value of the statistic $z$. For integer $\ell$, define $s_\ell$ as the school with the $\ell^{\text{th}}$ highest statistic value. Let $\bar{k} := \max\{k : \sum_{\ell \geq k} w_{s_\ell} \geq X/100\}$ and $\underline{k} := \min\{k : \sum_{\ell \leq k} w_{s_\ell} \geq Y/100\}$. The T$X$/B$Y$ ratio index is defined as

$$\frac{\sum_{\ell > \bar{k}} w_{s_\ell} z_{s_\ell} + \left(X/100 - \sum_{\ell > \bar{k}} w_{s_\ell}\right) z_{\bar{k}}}{\sum_{\ell < \underline{k}} w_{s_\ell} z_{s_\ell} + \left(Y/100 - \sum_{\ell < \underline{k}} w_{s_\ell}\right) z_{\underline{k}}}.$$

T20/B20 ratio is one of the main indexes used by EUROSTAT to report inequalities regarding member countries (European Commission, 2003), while T10/B50 and T10/B90 ratios have been used to measure changes to the income inequality over time (cf. Chancel and Piketty, 2021 and Bozio et al., 2024).

[29] This is a particular case of the generalization of the Gini index proposed by Donaldson and Weymark (1980), which is also equivalent to the Atkinson index when the society's inequality aversion parameter approaches infinity (see Appendix H for its details).

[30] Actually, we do not need replica economies; we only need the ratio of the numbers of non-participating teachers to participating teachers from each type at each school approaching infinity if the latter number is positive as well as a well-defined limit for the distribution of types of non-participating teachers.

[31] In the 8 subjects we consider in the empirical analysis, the set of non-participants represents 94.1%.

many applications.

Given a matching $\mu$ of the associated (re)assignment market, the value of a statistic $f$ may change when we vary $n$, as the statistic's input, the value distribution profile of matching $\mu$ takes into consideration the changing non-participating employee profile $E^n$. So we use $\delta_s^{\mu,n}$ for the eventual distribution of values induced by $\mu$ at school $s$ in the $n$-economy for a given $n$.

Given a statistic $f$, let the profile of value distributions $\delta^* = (\delta_s^*)_{s \in S}$ and the vector of the statistic outcome $z^{*f} := f(\delta^*)$ be defined by restricting our attention to only non-participants: for each school $s$, we define $\delta_s^{*\,v_k} := \frac{\left|E_s^{\theta_k}\right|}{|E_s|}$ for each $k$ and $\delta_s^* = (\delta_s^{*\,v_k})_{k=0}^{K-1}$.

For a given inequality index $\mathcal{I}$ and a statistic $f$, the economy is **generic** if $\mathcal{I}$ is continuously differentiable at $z^{*f}$ and all partial derivatives of $\mathcal{I}$ are non-zero at $z^{*f}$. Suppose $(L, H)$ is a partition of schools (i.e., $L \cup H = S$ and $L \cap H = \emptyset$). We say that a type ranking profile for schools $\trianglerighteq$ is **induced by partition** $(L, H)$ if each school in $L$ ranks types with higher type values ahead of types with lower values and each school in $H$ ranks lower value types ahead of higher value types with vacant-seat type $\theta_\emptyset$ ranked always below $\theta_0$. Formally, $\trianglerighteq$ satisfies the following:

- For each school $s \in L$, $\quad \theta_{K-1} \vartriangleright_s \theta_{K-2} \vartriangleright_s \ldots \vartriangleright_s \theta_1 \vartriangleright_s \theta_0 \vartriangleright_s \theta_\emptyset$.
- For each school $s \in H$, $\quad \theta_0 \vartriangleright_s \theta_\emptyset \vartriangleright_s \theta_1 \vartriangleright_s \theta_2 \vartriangleright_s \ldots \vartriangleright_s \theta_{K-1}$.

When $\mathcal{I}$ is differentiable at $z^{*f}$, we say $\trianglerighteq$ is the **natural type ranking profile of $\mathcal{I}$ and $f$** if it is induced by partition $(L^{\mathcal{I},f}, H^{\mathcal{I},f})$ where[32]

$$L^{\mathcal{I},f} = \left\{ s \in S \ : \ \frac{\partial \mathcal{I}}{\partial z_s}(z^{*f}) \leq 0 \right\} \quad \text{and} \quad H^{\mathcal{I},f} = \left\{ s \in S \ : \ \frac{\partial \mathcal{I}}{\partial z_s}(z^{*f}) > 0 \right\}.$$

We are now ready to state the first relation between status-quo improvement and inequality reduction for generic economies:

**Proposition 1.** *Consider an inequality index $\mathcal{I}$ and a continuous and LD-increasing statistic $f$. Fix a generic base economy with status-quo matching of the participants $\omega$. If in an $n$-economy with large enough $n$, a matching $\mu$ is status-quo improving for schools under the natural type ranking profile of $\mathcal{I}$ and $f$, then $\mathcal{I}\big(f(\delta^{\mu,n})\big) \leq \mathcal{I}\big(f(\delta^{\omega,n})\big)$.*

Before moving to the argument behind the above proposition, we make two comments. First, while the genericity of the base economy in Proposition 1 is very weak for standard indices such as the Gini index (for which it is enough that $z^{*f}$ has elements that are pairwise distinct), it is strong for indices such as T20/B20 for which the partial derivative of the inequality index in the base economy may be zero for many schools.

[32]Our analysis is insensitive to how we assign schools with zero index partial derivatives at $z^{*f}$ to these two sets, as for generic economies such schools do not exist, and for non-generic economies the results in the Appendix are insensitive to how these schools are assigned to these two sets.

However, assuming that the matching $\mu$ in Proposition 1 changes for at least one school with non-zero derivative, we show in Appendix A.7 how to extend the proposition to the environment where the inequality index is only assumed to be differentiable in the base economy.[33] Second, we assume that an inequality index does not depend on the market size $n$. However, since some indices depend on a vector of weights which itself depends on market sizes (e.g., the weighted Gini index mentioned in Section 4.1), these indices may depend on market sizes. To keep the notation concise, we do not model this in the main text. However, the above result easily extends to such indices, as clarified in Appendix A.3.

The proof of Proposition 1 is relegated to Appendix A.2 but we provide an intuition below. Before this, let us state the following intuitive lemma which is used in the proof.

**Lemma 1.** *Let $(L, H)$ be a partition of schools and $\trianglerighteq$ be its induced type ranking profile in an economy with status-quo matching $\omega$. Given an LD-increasing statistic $f$, for any status-quo improving matching $\mu$ for schools under $\trianglerighteq$, we have $f(\delta_s^\mu) - f(\delta_s^\omega) \geq 0$ for all $s \in L$, and $f(\delta_s^\mu) - f(\delta_s^\omega) \leq 0$ for all $s \in H$.*

The intuition behind Proposition 1 is as follows. In an $n$-economy, where $n$ is large enough, the number of teachers not participating greatly exceeds the number of participating teachers. In such a scenario, a change in $\mu$ has a relatively marginal impact on the profile of values of the statistic, and therefore, the relevant domain of $\mathcal{I}$ is relatively small.[34] Consequently, if the partial derivative $\frac{\partial \mathcal{I}}{\partial z_s}(z^{*f})$ is strictly negative (positive), it will remain so across the entire relevant domain of $\mathcal{I}$. According to Lemma 1, when moving to a status-quo improving matching, schools with negative (positive) derivatives experience an increase (decrease) in their statistic value. Consequently, the value of the inequality index $\mathcal{I}$ must decrease. Also note that for Lemma 1 to be true, the natural type ranking of each school in $H$ must rank the vacant seat type above the types of teachers with positive type value. This feature ensures that teachers with a positive type value cannot directly fill a vacant seat for schools in $H$. Indeed, if they were to do so, the statistic for a $H$ school could increase, which would, in turn, increase inequality. This natural type ranking used in SI-CC forbids any positive type value teacher to point to a school due to Improvement Condition 2.

It is crucial to note that this argument assumes an initial situation where there is a significant imbalance between participating and non-participating teachers, which, again,

[33] To address the issue of non-differentiability, a natural way to proceed in practice is to use a "perturbation" to the inequality index, ensuring that the perturbed index is continuously differentiable. This is always feasible by the the Stone-Weierstrass Approximation Theorem (Stone, 1937). One can then define the natural type ranking profile with respect to the perturbed index and obtain Proposition 1 for the nearby index. Of course, it is rare for the common inequality indices (and for common statistics) not to be differentiable at $z^{*f}$. For example, it would only be the case for the Gini or the T20/B20 indexes if two schools had exactly the same statistic value in the limit.

[34] Formally, for an $n$-economy, this relevant domain is $co\,\{f(\delta^{\mu,n}) : \mu \in \mathcal{M}\}$.

is a realistic assumption which holds in our main empirical application. However, without this assumption, the result may fail, and a status-quo improvement can exacerbate inequality. The underlying intuition is that a status-quo improvement can transform a school that initially lacks high-type teachers into one that is disproportionately assigned such teachers, largely surpassing in this dimension any other school. Hence, the school moves from a "poor" status to an "extremely rich" status in terms of high-type teachers, which may eventually exacerbate inequality. While this is an intriguing phenomenon, we consider it highly unlikely to occur in practical settings.[35]

Since SI-CC is status-quo improving under the type profile used, Proposition 1 applies to this mechanism, we obtain the following corollary:

**Corollary 1.** *Consider an inequality index $\mathcal{I}$ and a continuous and LD-increasing statistic $f$. Fix a generic base economy with the status-quo matching of participants $\omega$. Let $\mu$ be the matching obtained via the SI-CC mechanism induced by the natural type ranking profile of $\mathcal{I}$ and $f$ and any tiebreaker $\vdash$. Then, in an $n$-economy with large enough $n$, $\mathcal{I}\big(f(\delta^{\mu,n})\big) \leq \mathcal{I}\big(f(\delta^{\omega,n})\big)$.*

We illustrate our concepts introduced in Section 4, such as an $n$-economy, type value distribution under matchings, construction of a natural type ranking profile, and the implication of Corollary 1 in Example A.4 in the Appendix using the T20/B20 ratio inequality index and mean statistic. This example also illustrates the need for the large market assumption in Proposition 1 and Corollary 1.

Corollary 1 implies that the type ranking of schools can be designed in a way that SI-CC reduces inequality. However, this does not imply that SI-CC is the "optimal" mechanism for achieving this goal. There may be alternative mechanisms that outperform SI-CC in terms of inequality reduction, for instance, by violating status-quo improvement. However, we give two senses below in which this is not the case.

First, we prove that the converse of Proposition 1 holds in a large domain sense. Hence, there are economies in which a mechanism can reduce inequality only if it is status-quo improving for schools under the natural type ranking profile.

To state these results, we introduce a property of inequality indices: Let $\underline{s}$ be any school with the lowest value in $z$, i.e., $z_{\underline{s}} \leq z_s$ for each $s$. Similarly, let $\bar{s}$ be any school with the highest value in $z$, i.e., $z_{\bar{s}} \geq z_s$ for each $s$. Inequality index $\mathcal{I}$ is **regular** if, whenever $\mathcal{I}$ is differentiable at $z$, we have $\frac{\partial \mathcal{I}}{\partial z_{\underline{s}}}(z) < 0$ and $\frac{\partial \mathcal{I}}{\partial z_{\bar{s}}}(z) > 0$. This assumption is very weak and

[35] For our result to hold, large market is not critical: what matters is that, for each school $s$, the sign of the derivatives of the inequality index for $z_s$ remains the same on the relevant domain. Also, after a sequence of (re)assignments (occurring over several years)—assuming retirements etc, occur in a similar manner, we may reach a point where there may be two schools, one in $L$ and the other in $H$, which get so close to each other in terms of their values of statistic that the (re)assignment of teachers participating may be enough to have the school in $L$ moving to $H$ (and the other way around). However, the values are so close to each other that whether we put one of these schools in $L$ or in $H$ will only have a marginal impact on the value of the index.

satisfied by Gini and all standard inequality indices.[36]

**Proposition 2.** *Consider a regular inequality index $\mathcal{I}$ and a continuous and strictly LD-increasing statistic $f$. Then, there exists a generic base economy with a status-quo matching of the participants $\omega$, such that the following holds: in an $n$-economy with large enough $n$, an individually rational matching $\mu$ is status-quo improving for schools under the natural type ranking profile of $\mathcal{I}$ and $f$, if and only if $\mathcal{I}\big(f(\delta^{\mu,n})\big) \leq \mathcal{I}\big(f(\delta^{\omega,n})\big)$.*

We next prove that, for most inequality indices and statistics (including mean and median), no mechanism that is both individually rational and strategy-proof can generate less inequality than SI-CC using the natural type ranking profile $\trianglerighteq$. In order to state this result, we say that an inequality index $\mathcal{I}$ is **symmetric** if for each pair of schools $s, s' \in S$ such that $|E_s| + q_s = |E_{s'}| + q_{s'}$, for any $z, z'$ such that $z'_s = z_{s'}$, $z'_{s'} = z_s$, and $z'_{s''} = z_{s''}$ for each $s'' \in S \setminus \{s, s'\}$, we have $\mathcal{I}(z') = \mathcal{I}(z)$.

Consider two mechanisms $\varphi$ and $\psi$. Given an inequality index $\mathcal{I}$ and a statistic $f$, $\varphi$ has **less inequality than $\psi$ when possible** if, at every economy with the preference profile of the participating teachers $P$,

1. $\mathcal{I}\big(f(\delta^{\varphi(P)})\big) \leq \mathcal{I}\big(f(\delta^{\psi(P)})\big)$, and
2. whenever there is an individually rational matching $\mu$ such that $\mathcal{I}\big(f(\delta^{\mu})\big) < \mathcal{I}\big(f(\delta^{\psi(P)})\big)$, we have $\mathcal{I}\big(f(\delta^{\varphi(P)})\big) < \mathcal{I}\big(f(\delta^{\psi(P)})\big)$.

**Proposition 3.** *Consider a symmetric, regular, continuously differentiable inequality index $\mathcal{I}$ and a continuous and strictly LD-increasing statistic $f$. Then, there is no individually rational and strategy-proof mechanism that has less inequality when possible than the SI-CC mechanism induced by the natural type ranking profile of $\mathcal{I}$ and $f$ and any tiebreaker $\vdash$.*[37]

Several comments are in order. First, the proof uses a sequence of replica economies and we note that the argument works even when the size of the set of non-participating employees is arbitrarily large. Thus, it does hold in the context of Proposition 1 and so when SI-CC reduces inequality compared to the initial assignment. Second, assuming differentiability of $\mathcal{I}$ may exclude some inequality indices such as the T20/B20 or the Gini index, which are only differentiable almost everywhere. However, in Appendix A.8, we show that our result applies to the T20/B20, the Gini, and the maximin indices.[38] Finally,

[36]In Proposition A.2 of Appendix A.4, we prove that any (responsive) inequality index which satisfies the (strict) Pigou-Dalton transfer principle and scale invariance—two standard axioms—is regular. This implies that many inequality indices, including the Gini index, are regular. See for instance Idrees and Ahmad (2017) and Costa and Pérez-Duarte (2019) for surveys on inequality measures and their properties.

[37]Because, in Proposition 3, we impose the condition that the mechanism has less inequality *when possible* than SI-CC, the proposition does not exclude the possibility that there is an individually rational and strategy-proof mechanism which has a (weakly) smaller value of the inequality index than SI-CC at all instances and a strictly smaller value at some instance. However, this mechanism will sometimes fail to select a matching which strictly reduces inequality compared to SI-CC even though such a matching exists.

[38]It also applies to a large class of inequality indices which, in particular, are differentiable at any point which is strictly ordered—which is the case for all inequality indices we know of. See Remark 1.

we note that this result implies the following corollary of independent interest. Given an inequality index $\mathcal{I}$ and a statistic $f$, we say that a mechanism $\varphi$ **minimizes inequality** for $\mathcal{I}$ and $f$ if, for each economy with preference profile $P$, $\varphi(P) \in \arg\min \mathcal{I}\big(f(\delta^\mu)\big)$ where the minimum is taken over matchings $\mu$ that are individually rational. Since SI-CC induced by the natural type ranking does not minimize inequality (as made clear by the proof of Proposition 3), we obtain the following result.

**Corollary 2.** *Consider a symmetric, regular, differentiable inequality index $\mathcal{I}$ and a continuous strictly LD-increasing statistic $f$. Then, there is no strategy-proof mechanism that minimizes inequality for $\mathcal{I}$ and $f$.*

# 5 Empirical Analysis: The Case of France

## 5.1 Institutional Background

Teacher recruitment is centralized in France. Anyone who wishes to become a teacher has to pass a competitive examination. Those who succeed are assigned a teaching position by the Ministry for a probation period of one year, at the end of which they get tenure or not. Once public school teachers get tenure, they become civil servants. The government manages the first assignment of new teachers to a school and the transfer process of tenured teachers who previously received an assignment but wish to move.[39]

The assignment procedure takes place in two successive steps. Teachers are assigned to one of the 31 French regions in the first step using a centralized procedure. Tenured teachers who wish to change regions and new teachers submit a preference list over regions. In the second step, teachers who are newly assigned to a region and tenured teachers who wish to change schools within their region submit a preference list of schools. The same first-step mechanism is then applied to each region using the new inputs. Our empirical analysis focuses on the first step, regional assignment, due to the potential for strategic reporting of preferences during the second step.[40] From now on, we will treat each region as a large single school. Participation in the assignment mechanism is compulsory for all new teachers but optional for tenured teachers, who cannot be forced to change schools.

We combine data on 2013 (re)assignments with data on teacher and region characteristics (DEPP, 2013, 2014b). There were about 396,000 secondary public school teachers in France that year (DEPP, 2014a). When organizing the annual (re)assignment process, the central administration has to take into account not only a large pool of tenured teachers who wish to change positions but also some vacant positions that need to be filled—

[39] Our study focuses on the centralized (re)assignment process of public school teachers. Private schools employ 16% of teachers in France. This market is decentralized and has a separate exam.

[40] Preferences reported during the second step of the assignment are more difficult to interpret for two reasons. First, teachers can only rank up to 20 or 30 schools (depending on the region). Second, in addition to ranking schools, teachers can also rank larger geographic areas, such as cities, for instance.

about 9,800 public secondary school teachers retired in 2013—and new teachers, who have passed the recruitment exam, validated their probation year, and need to be assigned to their first jobs. In 2013, about 25,100 teachers participated in the regional (re)assignment process, including around 17,200 tenured teachers and 7,900 new teachers.

## 5.2 Data and Descriptive Statistics

We use data on the (re)assignment of teachers to one of the 25 out of (a total of) 31 French regions.[41] We have information on reported preferences and status-quo assignments of teachers, Ministry-mandated priorities of regions,[42] and the number of vacant positions in each region. We use *single* teachers from the eight largest subjects, such as French, Math, English, Sports, etc., and discard *couples* because they can submit joint preferences, which can be in different subjects and create dependencies across subjects. In order to keep the market structure balanced, we drop one seat for each teacher we omit.[43] Our final sample contains 10,483 teachers: 5,846 tenured teachers (55.8%) and 4,637 new teachers (44.2%). Table A.3 in Appendix B shows the decomposition for each subject and the number of vacant positions (3,912).[44] A central motivation of our analysis is to rebalance the unequal distribution of experienced teachers across regions. Part of this imbalance stems from differences in regions' attractiveness. Table A.1 reports descriptive statistics of teachers (Panel A), their status-quo assignment (Panel B), and the region they rank first (Panel C). Two regions surrounding Paris, called Créteil and Versailles, are particularly unattractive. The imbalance is blatant when we compare the number of teachers asking to leave the region and the number asking to enter. In Math, 56.2% of the tenured teachers who ask to change region come from Créteil or Versailles, but only 3.8% of tenured teachers rank one of these two regions as their first choice. The high number of transfer requests from less desirable regions leads to a significant exodus of teachers, creating numerous vacancies that must be filled. Every year, about 50% of the new teachers get their first assignment in one of the three least attractive regions (Créteil, Versailles, or Amiens). This structural imbalance is a major concern for policymakers and is often cited as a reason for the teaching profession's lack of appeal.[45]

[41] We discard the six overseas regions because of their specificities in terms of (i) teacher preferences—in contrast to what we find in our estimates, distance from the current location often becomes an attractive feature—and (ii) Ministry-mandated priorities—in some of these regions, like in Mayotte, teachers who grew up there get bonus points when they rank it first.

[42] The Ministry-mandated priorities are determined by a point system which is mainly based on teachers' experience, whether they ask for a spousal reunification in a region, and whether their status-quo school is disadvantaged. See Appendix F.2.1 for details.

[43] For each tenured teacher we discard, we drop her corresponding position. For new teachers, we find the share discarded (denoted by $K\%$), and we delete $K\%$ of vacant positions in each region.

[44] Every table, figure, and result indexed with prefix A is in the Appendix.

[45] In 2014, 24% of the positions for the recruitment exam in France remained vacant because of a shortage of applicants and the poor quality of those applying. In 2024, 17% of the positions remained vacant.

In addition, it creates large differences in the experience profile of teachers across regions. As reported in column (2) of Table A.2, in the youngest French region (Créteil), teachers have on average 10 years of experience, versus 15 years in the oldest region (Rennes), a gap that partly stems from the yearly inflow of new teachers to unattractive regions. Column (3) reports the share of teachers that have fewer than four years of experience—22.6% in Créteil, but only 5.5% in Rennes—a statistic policymakers care about because teachers contribute less to the educational development of their students during the initial years of their career (Bates et al., 2021, Chetty et al., 2014, and Rockoff, 2004). Reducing the uneven distribution of teachers by experience across regions and lowering the likelihood of new teachers being assigned to disadvantaged regions became key objectives for French policymakers (Cour des Comptes, 2013, 2017). They view these objectives as essential for closing the achievement gap and enhancing the long-term attractiveness of the teaching profession.

## 5.3 Specifications of the Empirical Analysis

**Teacher types.** We classify teachers into eleven experience bins, each representing a *teacher experience type*. The first five types correspond to teachers with zero, one, two, three, and four years of experience (types 1 to 5), respectively, while each subsequent bin groups teachers by four years of experience (types 6 to 11).[46] Vacant seats form a separate type, with the same experience value (zero) as the first bin of teachers (vacant seat type is designated as type 0), consistent with our theoretical model.[47] We use this experience partition because young teachers represent a large share of the participants (so dividing them makes sense), and the evidence on teacher value-added to attainment suggests an increasing value-added during the first few years of experience that flattens beyond these initial years (Bates et al., 2021, Chetty et al., 2014, and Rockoff, 2004). Figure A.2 shows a histogram of tenured and new teachers types in the (re)assignment market. Some new teachers have experience as they return to teaching after a break.

**Measuring inequality.** We use the T20/B20 ratio index introduced in Section 4.1 as our inequality index and the mean experience in a region as our statistic: We calculate the ratio of the mean experience of the top 20% of regions to the mean experience of the bottom 20% of regions, weighted by region size.[48] It is particularly useful for capturing dispersion between the tails of a distribution. The higher the T20/B20 ratio, the higher

[46] Figure A.3 provides a precise definition of the eleven experience types.

[47] As we already discussed, in practice, vacant positions are eventually filled by substitute teachers whom we consider comparable to the least experienced new teachers.

[48] Two things are important to note: First, if a region is partly in the bottom 20% (or top 20%), we include it fully when we discuss findings related to the bottom 20% regions (or top 20% regions) below. In calculating the T20/B20 index, this cutoff region is weighed by its proportional size that is inside the bottom (or top) 20%, but not its whole size (see Footnote 28). Second, when calculating teacher mean experience, we consider all teachers and vacant positions in a region, i.e., the teachers who request to move, those who do not request to move, and vacant seats.

the inequality. At status quo, this ratio is equal to 1.3588, meaning that the mean teacher experience in the T20 regions—those in the top 20% of the experience distribution—is 35.88% higher than the mean experience in the B20 regions—those in the bottom 20% of the distribution. Our empirical analysis will quantify by how much SI-CC reduces the T20/B20 ratio.[49]

**Regional type rankings for SI-CC.** We design type rankings that induce SI-CC to reflect the policymaker's distributional objectives. Our analysis in Section 4 guides us in this design. Using the T20/B20 ratio inequality index and the mean experience statistic, we construct an empirical version of the *natural type ranking profile* defined in Section 4. In each subject, regions are partitioned into two groups: $H$ and $L$. The regions in $H$ and $L$ are referred to as *high-type* and *low-type*, respectively. To build this partition, we compute the T20/B20 ratio and its partial derivative for each region at the status quo based on the experience of all teachers, i.e., those who request to move and participate in the (re)assignment process, those who do not, and vacant seats.[50] We then use the sign of the index partial derivative to create two groups: positive partial derivative regions are assigned to $H$, and negative partial derivative regions are assigned to $L$. There are also regions with a zero partial derivative. We assign those to $L$ if their mean experience is below the median region's and to $H$ otherwise.[51]

Recall that the natural type ranking for high-type regions orders (new) teachers with no experience (type 1) first, then the vacant seat type (type 0), followed by the other teacher types ordered by increasing experience (i.e., types are ranked as 1,0,2,3,...,11). In contrast, low-type regions' natural type ranking orders teacher types by decreasing level of experience and the vacant seat type last (i.e., types are ranked as 11,10,...,1,0).[52] Our ultimate goal in designing type rankings in this manner is to ensure that the outcome of

[49] We also present robustness checks in Appendix H using an alternative index, the maximin inequality index, and an alternative statistic, the share of teachers with more than four years of experience in a region.

[50] In Section 4, we used the limit of replica economies of non-participants to calculate the initial mean experience statistic vector. As a result, this statistic vector ignores all participating teachers in the (re)assignment process, as they become negligible in the limit. In reality, our economy is large but finite, so we include these teachers in our empirical construction. Excluding them to define the natural type ranking does not change qualitatively the results.

[51] As made clear in the proof of Proposition A.3, our theoretical analysis is robust to which set we assign the zero-partial derivative regions. In the limit of replica economies, their contribution to inequality is negligible. Here, in our large but finite economy, we pay more attention to the construction, and we perform robustness checks in which we simulate what happens if we assign all these regions to $L$ or to $H$, respectively. Our results are similar. See Appendix I for details.

[52] Table A.2 reports statistics on region types across the eight subjects. Column (1) reports the share of subjects in which a region is classified as low-type. The three youngest regions, Créteil, Versailles, and Amiens, are low-type in all subjects (and they are the largest regions with 20.8% of the whole market; see Column 4). Due to their large size, the three youngest regions correspond to the B20 regions; except in one subject, Amiens is not in B20 (and only the other two comprise the B20 regions), and in three subjects, there are additional regions in B20. On the other hand, the T20 regions are smaller. Columns (2) and (3), which report teacher mean experience at status quo and the share of teachers with fewer than four years of experience, reveal a large correlation between region low/high type and these two statistics.

our mechanism yields a more equal distribution of teachers across regions, as desired by the French Ministry of Education.[53] SI-CC can achieve this goal as proven in Proposition 1 and Corollary 1 (See also Proposition A.3) in a sufficiently large economy. This finding remains to be verified empirically, as is the magnitude of a reduction of inequality.

**Mechanisms.** Our counterfactual analysis aims to formally define possible inputs, quantify the performance of SI-CC, and benchmark its performance with two mechanisms: a variant of a widely studied mechanism and a practically relevant mechanism:

- **TTC*:** This is a variant of SI-CC that relaxes the mechanism features that ensure status-quo improvement for regions. More precisely, this mechanism differs from SI-CC in two aspects in its algorithmic description: (1) we lift the restrictions on the set of regions to which a teacher can point, and (2) tenured teachers can now start a chain and potentially leave their position without being replaced (see Appendix F for a formal definition). This benchmark is similar to the well-known TTC-variant mechanism "You request my house – I get your turn" (YRMH-IGYT) introduced by Abdulkadiroğlu and Sönmez (1999). TTC* is strategy-proof, Pareto efficient, and individually rational for teachers, but not status-quo improving for regions. Intuitively, TTC* is expected to generate higher teacher welfare and more mobility than SI-CC at the cost of a potentially more unequal teacher distribution (see Section 5.4).
- **The Current French Mechanism:** This mechanism uses Gale and Shapley (1962)'s teacher-proposing deferred acceptance (DA) algorithm with the modification to regions' priorities that each tenured teacher is moved to the top of their status-quo region's priorities (priorities are otherwise respected within status-quo teachers and within other teachers, respectively). This modification ensures individual rationality for tenured teachers. We use the Ministry-mandated priorities for regions which makes this mechanism equivalent to each step of the current French assignment process. This mechanism provides a second interesting benchmark that does not satisfy status-quo improvement but may result in higher mobility for tenured teachers, who generally receive higher priority than new teachers in all regions. As the mechanism is designed to satisfy "no justified envy" among non-status-quo employees of regions (see Section 5.5), it assigns high experienced tenured teachers to more desirable regions, who generally have higher priority than less experienced teachers.[54]

**Estimation of teacher preferences.** As we just mentioned, the Ministry uses a modi-

[53]Evidence from the US also suggests that younger regions value teacher experience more than older regions (Bates et al., 2021). Title I school principals have a stronger preference for high-value-added teachers (which strongly correlates with more experienced teachers) than principals in non-Title I schools, similar to our natural type ranking profile.

[54]A matching $\mu$ satisfies "no justified envy" among non-status-quo teachers if for any teacher $t$ and region $s \neq \omega_t$ such that $t$ prefers $s$ to her assignment $\mu_t$, there is no teacher $t' \in \mu_s \setminus \omega_s$ such that the region $s$ gives higher priority to $t$ than $t'$.

fied version of the DA algorithm to assign teachers to regions using the Ministry's known, mandated priorities. Teachers can rank all regions when they submit their preference list. Yet, even under strategy-proof mechanisms, experimental and empirical papers show that truthfulness is a strong assumption (Chen and Sönmez, 2006, Pais and Pinter, 2008, Rees-Jones, 2018, Chen and Pereyra, 2019, and Hassidim et al., 2017). In our context, French teachers have reasonably accurate information on their admission probabilities to each region, which might encourage some to discard from their preference list the regions with a low assignment chance.[55] To avoid the potential bias generated by teachers omitting regions, instead of using the reported preferences, we estimate teacher preferences under a *stability assumption* developed by Fack et al. (2019) and applied to the teacher assignment by Combe et al. (2022). Appendix G provides a detailed presentation of the estimation method and reports results on the preferences of tenured and new teachers. After estimating teacher preferences, we use our utility estimates to draw teacher preferences 1,000 times. In each of the eight subjects and for each draw, we use these simulated preferences to run the mechanisms. The results reported in the next section correspond to averages over these 1,000 draws, aggregated over the eight subjects.

## 5.4 Benefits and Costs of Status-quo Improvement Property

This section inspects the benefits and costs of requiring status-quo improvement in designing assignment mechanisms. To do so, we compare SI-CC to TTC*, which is not a school-status-quo improving mechanism.

**Better distribution of teacher experience**. For the (re)assignment market, the left panel of Figure 1 shows the cumulative distribution of teacher experience at SI-CC, TTC*, and status-quo matchings in B20 regions.[56] Every year, many experienced teachers leave these regions. They are mostly replaced by inexperienced teachers. This imbalance hints at a preference for tenured teachers for other older regions. As a result, the outcome under TTC* is unlikely to improve over the status quo in B20 regions. Our analysis confirms this. The distribution of teacher experience under TTC* does not Lorenz dominate the status quo in B20 regions, while it does under SI-CC (see the left panel of Figure 1).[57]

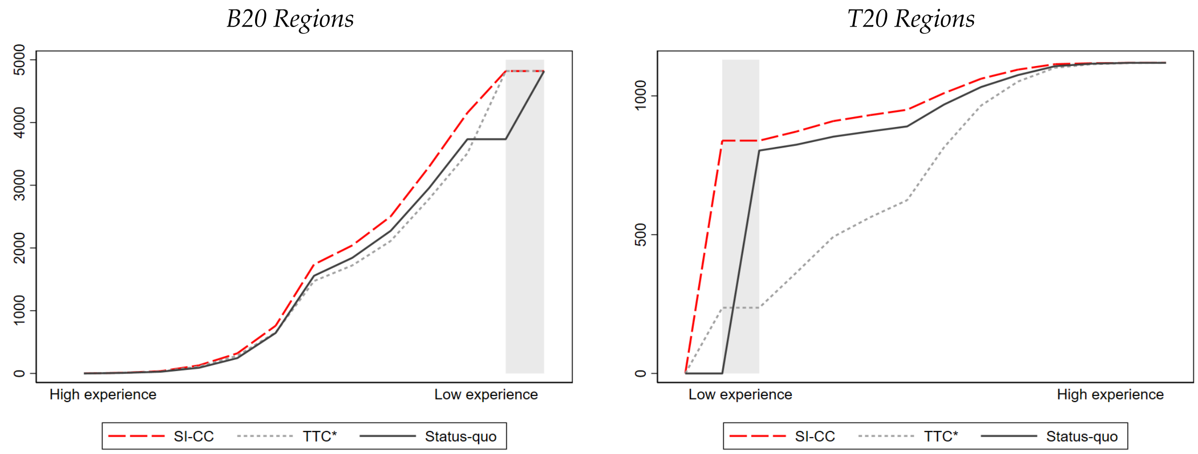
**Fact 1.** *In the B20 regions of France, the distribution of teacher experience (accounting for participants, non-participants, and vacant seats) under SI-CC Lorenz dominates that under TTC*.*

[55] Cutoffs for entry in each region are published every year. Combe et al. (2022) show that these cutoffs are relatively persistent over time, providing reasonably accurate information to teachers on their chances of assignment to each region.

[56] See Table A.2 for the status-quo experience distribution in all regions and Figure A.4 for the cumulative distributions that include non-participating teachers. Stochastic dominance across mechanisms is identical when we account for non-participants, as the cumulative distributions for each mechanism and for the status-quo matching are all shifted up by the same curve pertaining to the non-participating teachers.

[57] Figure A.5 shows that the distributional performance of SI-CC is unaffected by the choice of chain-selection tiebreaker rule.

**Figure 1:** Cumulative Distribution of Teacher Experience Types in the (Re)assignment Market




Notes: This figure shows the cumulative distribution of teacher experience types in the (re)assignment market in the B20 regions (left) and in T20 regions (right) under SI-CC, TTC*, and status quo. We identify T20 and B20 regions in each subject and find the cumulative distribution aggregated across subjects. The horizontal axis reports the eleven experience types of teachers, ordered from most experienced to least experienced (left panel) and from least experienced to most experienced (right panel) in accordance with the natural type rankings of these regions. The area shaded in gray corresponds to vacant positions. As B20 and T20 regions are determined with respect to all participating and non-participating teachers, their total participating teacher numbers are different. T20 regions have fewer participating teachers.

*Moreover, SI-CC assigns 1,527 teachers with zero or one year of experience to these regions, while TTC* assigns 2,039 of them.*

In the T20 regions, the objective is to assign younger teachers to reduce overall inequality (see the right panel of Figure 1 for the (re)assignment market).

**Fact 2.** *In the T20 regions of France, the distribution of teacher experience (accounting for participants, non-participants, and vacant seats) under SI-CC Lorenz dominates that under TTC*. Moreover, SI-CC assigns 871 teachers with zero or one year of experience to these regions, while TTC* only assigns 363 of them.*

The difference in performance between SI-CC and TTC* stems from the following mechanics. Due to the status-quo improvement property, SI-CC prevents many tenured teachers from leaving B20 regions, as few high-experience teachers want to come there from T20 regions. This limits the possibility of assigning high-experience teachers to the attractive T20 regions under SI-CC but not under TTC*. These teachers flow to the vacant seats of T20 regions through chains under TTC*. Thus, in aggregate, very few new teachers with no experience are assigned to T20 regions under TTC*. On the other hand, under SI-CC, new teachers with no experience are assigned to T20 regions through short chains to fill vacant seats. This mechanics also explains why the experience distribution under TTC* in T20 regions does not Lorenz dominate the status quo distribution (unlike SI-CC). Moreover, under SI-CC, due to the status-quo improvement property, new teachers with higher experience are primarily assigned to B20 regions to increase these regions' experience.

**Lower inequality among regions.** The status-quo improvement property ensures that regions are not "negatively affected" by the (re)assignment of teachers in terms of their natural type rankings. *Older regions* (which is used synonymously with high-type regions in what follows) become relatively younger, and *younger regions* (which is used synonymously with low-type regions) become relatively older, reducing the initial difference in teacher experience among regions. While the previous paragraphs discussed the distributional performance of the mechanisms for the B20 and T20 regions, we now consider their performance across all regions. Figure 2 plots, for each region, the change in teacher experience between the two mechanisms we simulate and the status-quo matching.

In the (re)assignment market, compared to the status-quo matching, SI-CC increases the average experience of teachers by 2.1 years in the younger regions and reduces the average experience of teachers by 0.7 years in the older regions (see the left panel of Figure 2). As a result, SI-CC effectively lowers the experience difference between younger and older regions by about 2.8 years. We reach similar conclusions regarding SI-CC's performance when considering the change in experience for the entire market, including teachers who do not participate in the (re)assignment process (see the right panel of Figure 2).

Figure 2 also confirms the poor performance of TTC* in terms of distribution compared to the status quo. In the (re)assignment market, TTC* reduces the average teacher experience in the three youngest regions (which primarily make up the B20 regions) by 0.26 years. It also increases the average experience in older regions by 1.5 years. We summarize these findings as follows:

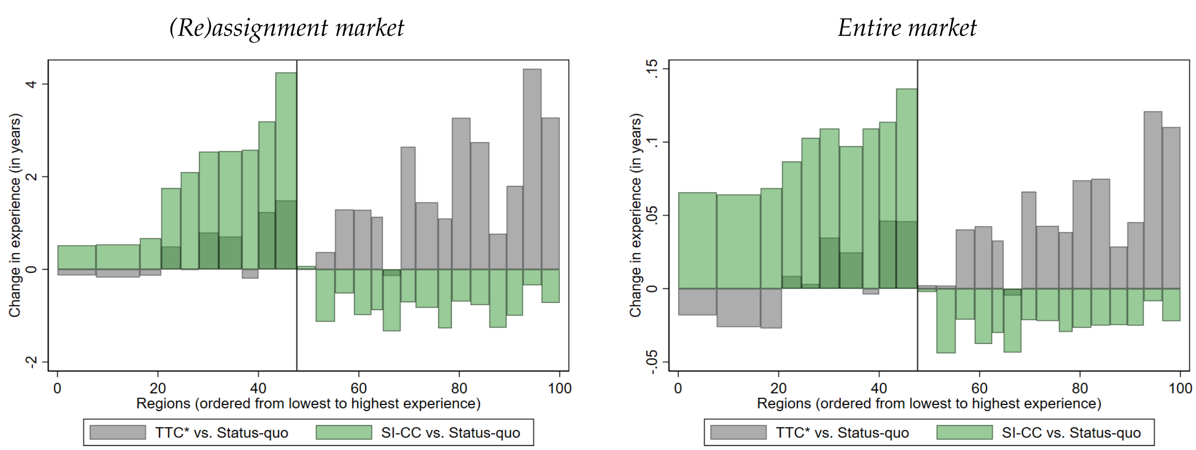
**Fact 3.** *SI-CC reduces the large gap in average teacher experience that exists at the status-quo matching between the younger regions and the older regions. Among teachers who participate in the (re)assignment process, this gap goes down by 2.8 years of experience. SI-CC also reduces the gap by 3.9 years compared to TTC*.*

In the entire market — comprising both participating and non-participating teachers, as well as vacant positions — the more balanced distribution of teacher experience is reflected in the improvement of our inequality index (see Table 1):

**Fact 4.** *The improved distribution of teacher experience for the entire market under SI-CC is reflected by a reduction of the T20/B20 ratio (1.3487) compared to TTC* (1.3691). SI-CC also leads to a lower T20/B20 ratio than the status quo (1.3588), but TTC* does not. These correspond to a 2.79% drop of the 35.88% status quo experience difference for SI-CC and a 2.87% surge for TTC*.*

Thus, while under TTC* the average teacher experience in the T20 regions is 36.91% higher than the average experience in the B20 regions, the experience gap ratio decreases to 34.87% under SI-CC. The reduction in the T20/B20 inequality index achieved by SI-CC aligns with our theoretical results (i.e., Proposition 1 and Proposition A.3). We interpret this as evidence that the French teacher market is sufficiently large for our findings to hold.

**Figure 2:** Change in Teacher Experience (in years)




Notes: This figure plots the change in teacher experience (in years) between TTC* and the status quo (grey bars) and between SI-CC and status quo (green bars). First, for each subject, we compute the average experience of teachers at each region at the status quo, including teachers who request to move and those who do not. We rank regions in the increasing order of experience. For each subject, we find the difference in teacher experience for the (re)assignment market and the entire market under each mechanism from the status quo—also plotted in Figures A.7 and A.8 in the Appendix using the region weights as the size of the X-axis bin of each rank (defined as the number of teachers and the number of vacant positions in the region divided by the national total for the subject). Finally, starting with the least experienced regions and averaging over subjects with their respective weights, we plot the experience gap (on the Y-axis) and the average region weight (on the X-axis), and repeat the exercise for the second least experienced regions, third, and so on.

**Trade-off between teacher distribution and teacher welfare.** We now examine whether SI-CC's better distributional performance comes at the expense of teacher welfare. Surprisingly, Table 1 reveals that, on average, teachers prefer the outcomes of SI-CC over TTC*. Thus, in this dimension, the distributional constraint aimed at reducing inequality does not compromise overall teacher welfare. However, this aggregate performance of SI-CC over TTC* masks notable differences among teachers. Indeed, new teachers benefit more under SI-CC, while tenured teachers fare better under TTC*. Specifically, the rank distribution of regions assigned to new teachers under TTC* is Lorenz-dominated by the distribution under SI-CC, as shown in Panel D of Table 1 for the first four choices. Conversely, tenured teachers experience the opposite pattern. These findings are summarized in the following fact.

**Fact 5.** *On average, teachers prefer the region they are assigned under SI-CC ($9.5^{th}$ ranked region) compared to TTC* ($9.7^{th}$ ranked region). On average, new teachers prefer the region they are assigned under SI-CC ($11.5^{th}$ ranked region) compared to TTC* ($13.7^{th}$ ranked region). The* *distribution of the region ranks that new teachers are assigned to under SI-CC Lorenz dominates that under TTC* up to rank 22 (over 26). For tenured teachers, the distribution of the region ranks under TTC* Lorenz dominates that under SICC.*[58]

These results, reported in Panel E of Table 1, are important for two reasons: first, they reveal that the welfare costs of SI-CC's improved distributional performance are less evident than anticipated; and second, since satisfying new teacher preferences is a key factor in the attractiveness of the teaching profession (Cour des Comptes, 2013, 2017), the positive outcome for new teachers could be viewed as a potential argument in favor of SI-CC.

These differences in rank distributions between new teachers and tenured teachers can be understood as follows. SI-CC's better performance for new teachers is due to a much larger number of tenured teachers leaving the B20 regions under TTC* (798) than under SI-CC (172). These exiting tenured teachers have to be replaced, and new teachers are the most likely substitutes due to lower demand from other tenured teachers. In our counterfactual analysis, we see that 1,779 new teachers are assigned to the B20 regions under TTC* versus 1,110 under SI-CC.

The lower teacher mobility under SI-CC explains the worse distribution of ranks of tenured teachers under SI-CC. Indeed, more tenured teachers move to a new region under TTC* (2,040 teachers) compared to SI-CC (986 teachers). Teacher mobility costs are higher in younger regions, in particular in the B20 regions, than in older regions and in the T20 regions. While similar numbers of tenured teachers leave older regions under SI-CC and TTC* (477 vs. 672), more tenured teachers leave younger regions under TTC* (1,369, with 798 from the B20 regions) than under SI-CC (509, with 172 from the B20 regions). The lower demand from tenured teachers for the younger regions compared to the demand for the older regions explains the large difference in outflow under TTC*. The status-quo improvement requirement under SI-CC limits outflow from younger regions, a concern explored further in Section 5.5.

## 5.5 Increasing Mobility from Young Regions

The results we have presented so far show that SI-CC's superior distributional performance (compared to its benchmark mechanism, TTC*) comes at the cost of reduced mobility, particularly for employees in the B20 regions. In this section, we demonstrate that this cost can be mitigated with a simple modification of SI-CC, which we refer to as **SI-CC***. This mechanism adjusts new teachers' preferences by placing the three youngest regions (Créteil, Versailles, and Amiens)—which primarily comprise the B20 regions (see Footnote 52)—at the top. Aside from this adjustment, the original ranking is preserved

[58] Table 1 shows the stochastic dominance for the first four ranks. In addition, we formally test whether the stochastic dominance is statistically significant for the entire rank distribution. This allows us to determine the rank up to which stochastic dominance applies.

**Table 1:** Teacher Welfare

| | Main mechanisms | | Other mechanisms | |
|---|---|---|---|---|
| | SI-CC | TTC* | SI-CC* | Current French |
| | (1) | (2) | (3) | (4) |
| **Panel A. Inequality Index** | | | | |
| Ratio T20/B20<br>*Value at status-quo = 1.3588* | 1.3487 | 1.3691 | 1.3489 | 1.3829 |
| **Panel B. Teacher mobility** | | | | |
| Tenured teachers moved and new teachers assigned | 4,910 | 5,964 | 5,226 | 5,635 |
| Tenured teachers moved - from the B20 regions | 172 | 798 | 537 | 910 |
| Tenured teachers moved - from the T20 regions | 152 | 203 | 152 | 114 |
| Tenured teachers moved - from all regions | 986 | 2,040 | 1,307 | 1,711 |
| New teachers assigned | 3,924 | 3,924 | 3,919 | 3,924 |
| New teachers unassigned | 713 | 713 | 718 | 713 |
| New teachers with 0 exp. assigned - to the B20 regions | 665 | 1,311 | 987 | 1,406 |
| New teachers with exp. $>0$ assigned - to the B20 regions | 445 | 468 | 568 | 455 |
| **Panel C. Cumulative distribution of ranks of regions that tenured teachers are assigned** | | | | |
| Rank $=1$ | 295 | 842 | 341 | 675 |
| Rank $\leq 2$ | 1,025 | 1,795 | 1,137 | 1,515 |
| Rank $\leq 3$ | 1,496 | 2,307 | 1,627 | 1,980 |
| Rank $\leq 4$ | 1,865 | 2,624 | 2,008 | 2,332 |
| Rank any | 5,846 | 5,846 | 5,846 | 5,846 |
| **Panel D. Cumulative distribution of ranks of regions that new teachers are assigned** | | | | |
| Rank $=1$ | 812 | 467 | 692 | 315 |
| Rank $\leq 2$ | 1,177 | 733 | 986 | 511 |
| Rank $\leq 3$ | 1,427 | 916 | 1,175 | 671 |
| Rank $\leq 4$ | 1,626 | 1,068 | 1,323 | 814 |
| Rank any | 3,924 | 3,924 | 3,919 | 3,924 |
| **Panel E. Average rank of region assigned** | | | | |
| All teachers | 9.5 | 9.7 | 10.1 | 10.4 |
| Tenured teachers | 7.8 | 6.5 | 7.5 | 6.9 |
| New teachers | 11.5 | 13.7 | 13.3 | 14.8 |

Notes: Panel A of this table reports the T20/B20 ratio calculated for each allocation. Panel B reports statistics on teacher mobility: numbers of tenured teachers who moved to a new region and assigned and unassigned new teachers. Panels C and D present the cumulative distribution of the ranks of the regions teachers are assigned to in their preferences. Panel E reports statistics on the average rank of the region teachers obtain. The average numbers in the counterfactual simulations are rounded to the nearest integer.

within the three youngest regions and among the other regions. We prove in Appendix F.3 that SI-CC* is strategy-proof.[59] Moreover, it is status-quo improving for schools. By Proposition 1 (and Proposition A.3), in large markets, it reduces inequality when using natural type rankings, as we do here.

The Ministry's current priorities are primarily based on experience, so new teachers, who generally have less experience than tenured teachers, have low priority in all regions. Even when they rank regions that are not among the youngest (and more attractive) at the top of their preferences, they are unlikely to be assigned to one of these regions due to the "no justified envy" feature of the current mechanism: many tenured teachers also rank these regions highly, and their demand exceeds these regions' quotas. Thus, the current mechanism, which uses the DA algorithm, implicitly forces new teachers to go down their preferences until they apply to the youngest regions. Moreover, the French Ministry of Education completes any incomplete rank-order list of new teachers so that all regions are ranked to ensure all vacancies are filled. Therefore, requiring new teachers to rank the youngest regions at the top of their preferences enables SI-CC* to reflect the current treatment of new teachers by policymakers.

**Benchmark mechanisms for SI-CC*.** There are two natural benchmarks to which we compare the outcome of SI-CC*, namely SI-CC (without preference modification for new teachers) and the Current French mechanism (which uses priorities that are determined by the Ministry's formula, see Appendix F.2.1). These benchmarks allow us to check whether SI-CC* can increase the mobility of tenured teachers closer to the Current French mechanism than what SI-CC achieves while maintaining a better distribution of teachers (closer to SI-CC) than the Current French mechanism.

**Counterfactual results.** Several findings stand out in the results reported in Table 1. First, under SI-CC*, significantly more tenured teachers move away from the B20 regions than under SI-CC. Mobility from these regions goes up from 172 under SI-CC to 537 under SI-CC*. Second, as expected, SI-CC* leads to a smaller improvement in the distribution of teachers. However, inequality still reduces below the status-quo level. While SI-CC and SI-CC* produce almost identical distributions in the T20 regions, a slight difference emerges in the B20 regions (see Figure 3). SI-CC* assigns 97 more teachers with zero or one year of experience to the B20 regions than SI-CC. As a result, SI-CC* leads to a reduction in the average teacher experience in the B20 regions compared to SI-CC (See Figure 4). However, it is interesting to note that the 365 additional teachers who leave the B20 regions under SI-CC* (compared to SI-CC) are partly replaced by experienced new teachers (123 more under SI-CC* compared to SI-CC, see Table 1). Hence, the experience

[59] It is also individually rational if the three youngest regions are assumed acceptable to new teachers, a plausible assumption given that the Ministry of Education currently completes new teachers' lists to rank all regions and fill vacancies. Dropout rates are negligible even though a majority of new teachers are assigned to the youngest and least attractive regions.

gap does not go up much under SI-CC*, and inequality still goes down below the status quo.

Finally, for all distributional metrics considered, SI-CC* not only has a much better performance than the status quo but also, and perhaps most importantly, than the Current French mechanism. As noted before, the average teacher experience in the T20 regions is 35.88% higher than in the B20 regions at status quo (in the entire market). SI-CC* (resp. SI-CC) decreases this experience gap ratio by 2.75% (resp. 2.79%), whereas the Current French mechanism increases it by 6.72%. In both younger and older regions, the cumulative distribution of teacher experience under SI-CC* Lorenz dominates the status quo and the Current French mechanism (see Figure 3).[60] As a result, SI-CC* better fulfills the twofold objective of (i) making younger regions older and (ii) making older regions younger (See Figure 4). These latter results are important as they show that there is not only significant room to improve upon the status quo but also upon the allocation that the Ministry of Education reaches after the annual (re)assignment process. This latter improvement comes at a small cost regarding the overall teacher mobility, i.e., 5,635 teachers move under the Current French mechanism versus 5,226 under SI-CC*. Among the tenured teachers of the B20 regions, 910 teachers move away under the Current French mechanism versus 537 under SI-CC* (as a reminder, 798 teachers move away under TTC*). However, on average, teachers prefer their assigned region under SI-CC* (10.1$^{\text{th}}$ rank) to the Current French mechanism (10.4$^{\text{th}}$ rank). Finally, SI-CC* performs much better than the current French mechanism when it comes to limiting the number of inexperienced new teachers assigned one of the B20 regions (987 under SI-CC* versus 1,406 under the current French mechanism).

# 6 Related Literature

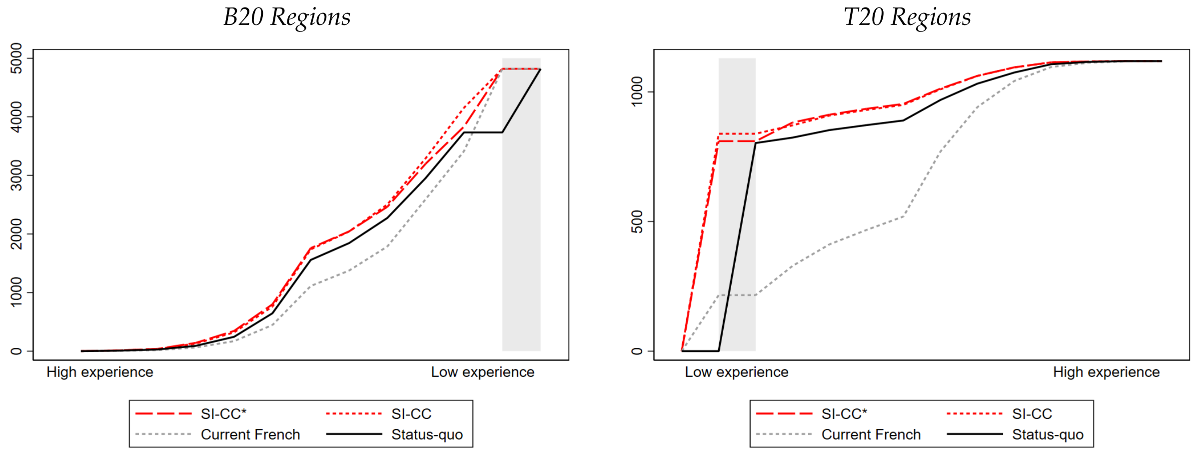
The SI-CC mechanism has its roots in the top-trading cycles algorithms (see Shapley and Scarf, 1974, Abdulkadiroğlu and Sönmez, 1999, Pápai, 2000, Roth et al., 2004 and Dur and Ünver, 2019). On the other hand, the design of such mechanisms or the utilized priorities based on the formal theory of inequality measures (or social welfare functions) has no antecedent in market design, as far as we know. Thus, we discuss some related literature on the design of constrained efficient mechanisms in allocation problems.

The design of efficient mechanisms in two-sided matching markets with a balanced exchange constraint was previously studied by Dur and Ünver (2019) in the context of student and worker exchange programs. The main difference from the current model is that status-quo improvement was not required in the previous paper. Status-quo improvement substantially changes modeling choices and mechanism design.

Our paper is also related to Combe et al. (2022) who study teacher reassignment, in-

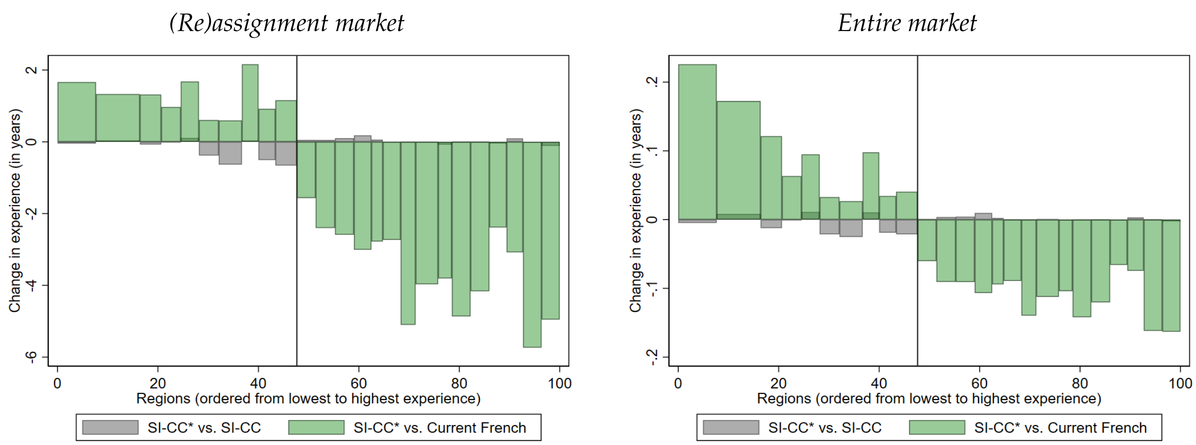
[60] See Figure A.6 for the entire market.

**Figure 3:** Cumulative Distribution of Teacher Experience in the (Re)assignment Market - SI-CC* vs. SI-CC and Current French




Notes: See the caption of Figure 1 for the construction methodology of this figure.

**Figure 4:** Change in Teacher Experience (in years) - SI-CC* vs. SI-CC and Current French




Notes: See the caption of Figure 2 for the construction methodology of this figure.

troduce a class of mechanisms that is two-sided Pareto efficient, and show that a unique selection in this class is teacher optimal. Although both papers consider teacher reassignment problems, our paper differs in important respects. Most importantly, the mechanisms introduced in this previous study do not embed distributional objectives. Their paper focuses only on Pareto-efficiency-based design and efficiency gains, which stands in stark contrast with the objective of our paper, namely to design mechanisms that lead to a better distribution of workers while satisfying incentive and constrained efficiency properties using a formal approach based on an inequality index. Hence, one of our main methodological contributions connecting status-quo improvement with inequality reduction is orthogonal to Combe et al. (2022)'s contributions. Also this previous study largely focuses on teachers with a status-quo assignment and largely ignores new teachers. Accounting for the entire market, as we do in this paper, is important because new teachers are a key driver of the unequal distribution of teachers in schools. Additionally, many of the desirable mechanism properties, such as status-quo improvement, do not easily translate to markets with vacancies and new teachers, and as a result theoretical and conceptual treatment in our paper is substantially more complex along this dimension.[61]

The study of efficient mechanisms under distributional constraints remain underexplored despite prior studies, including Suzuki et al. (2018) and Hafalir et al. (2024). These works establish sufficient conditions for a variant of TTC that incorporates constraints and satisfies desirable properties, relying on $M^{\natural}$-concavity of the policy function. Our ultimate objective is to reduce inequality when the policy function is an inequality index which is typically not $M^{\natural}$-concave. Under a large market assumption, we demonstrate the feasibility of constructing efficient mechanisms with desirable properties, even under complex (yet standard) distributional constraints.

The SI-CC mechanism is also related to *improvement cycles* approach introduced by Erdil and Ergin (2008, 2017). These two papers introduce algorithms to achieve a constrained-efficient (or Pareto-efficient, respectively) stable matching in a school-choice problem (or two-sided matching market) with indifferences in preferences of participants starting from an arbitrary envy-free, individually rational, and non-wasteful (or stable) one. They implement a stable improvement cycle in every round, making the students (or both sides of the market) better off. In this sense, especially Erdil and Ergin (2017) algorithm is similar in improving agents subject to some additional axiom. In SI-CC, this axiom is status-quo improvement, while it is stability in the other two papers. While we have strict preferences, the whole goal of other papers is achieving efficiency goals under weak preferences, as we already have deferred acceptance algorithms of Gale and Shapley (1962) that achieve these goals under stability with strict preferences. Therefore, when used as direct mechanisms, these algorithms are not strategy-proof (or not strategy-proof for either side of the market), while SI-CC is.

On the empirical side, our paper also complements a fast-growing literature that explores wage-based solutions to the unequal distribution of quality teachers in schools. Several recent papers have developed equilibrium models of the labor market for teachers, and used these models to inspect the effect of compensation policies on the distribution of teacher quality (Biasi et al., 2021, Bobba et al., 2021, Bates et al., 2021, and Tincani, 2021). Despite the tremendous progress made by these papers to shed light on price-based solutions to distributional concerns, much less is known on solutions for labor markets that do

[61] Note that, even in the pure reassignment market, which is a special case of our entire market, we show that our SI-CC mechanism does not belong to the class of TO-BE mechanisms defined in Combe et al. (2022), and vice-versa. (see Example A.3 in Appendix D).

not rely on prices, or that do so imperfectly. Yet, several countries use a centralized process to assign teachers to schools, like Germany, Italy (Barbieri et al., 2011), Turkey (Dur and Kesten, 2019), Mexico (Pereyra, 2013), Peru (Bobba et al., 2021), Uruguay (Vegas et al., 2006), Portugal, and the Czech Republic (Cechlárová et al., 2015).[62] Understanding how to address distributional concerns in these regulated markets is important. The evidence also points to a large cost of wage-based policies to attract good teachers in priority education schools, which might encourage countries to rely on more centralized solutions (Bobba et al., 2021).[63] A more recent empirical paper, Laverde et al. (2023), quantifies the equitability and gains in student learning outcomes that can be achieved through a more efficient and equitable centralized (re)assignment process of teachers over the current semi-centralized scheme used in Minnesota in the US.

Finally, our paper builds on a recent literature developing demand estimation methods in school choice environments (Abdulkadiroğlu et al., 2017, Agarwal and Somaini, 2018, and Calsamiglia et al., 2020). In particular, we build on techniques based on discrete choice models with personalized choice sets which are relevant for preference estimation when reported preferences might fail to be truthful even under strategy-proof mechanisms (Fack et al., 2019, Akyol and Krishna, 2017, and Artemov et al., 2019).

# 7 Concluding Remarks

**Minimalist Market Design.** In our main application, we identified inequality reduction across regions in France and efficiency as key objectives of the French Ministry of Education. Accordingly, our goal is to develop mechanisms that can efficiently and individually rationally (re)assign teachers to schools while reducing inequality. We have demonstrated that the current mechanism used in France falls short of achieving this objective. In line with the 'Minimalist Market Design' paradigm proposed by Sönmez (2023), we address these shortcomings by proposing a new transparent mechanism that involves minimal interference with the existing practices employed by the Ministry.

Our approach suggests a way for the Ministry to redesign teachers' priorities and conduct the assignment process like the current practice. In fact, a mechanism similar to the TTC and our proposal, which allows teachers to swap positions, is already in use for primary school teacher assignments in France.[64] In this mechanism, as in our proposal,

[62]Beside these fully centralized markets, in most teacher labor markets (like in the US), wage variations are strongly limited by rigid pay scales that determine teacher salary as a function of experience. Biasi et al. (2021) provides insightful discussions on non-flexible wage policies in the US: "Most US public school districts pay teachers according to *steps-and-lanes* schedules, which express a teacher's salary as a function of their experience and education."

[63]Bobba et al. (2021) finds that "it would take six times the current budget to equalize access to teacher quality across Peru". Thakur (2020) also investigates the distributional consequences of centralized assignment for Indian Administrative Service jobs, the top-tier government jobs located across the country before and after a mechanism change.

[64]In essence, this consists of running the TTC with the outcome of the Current French mechanism as the

there are school/region priorities that are set by the Ministry. Thus, our proposed mechanism offers a straightforward recommendation that does not require radical philosophical and administrative changes. We believe that this minimal interference with the current practices increases the likelihood of our suggested mechanisms (SI-CC or SI-CC*) being adopted by the authorities in charge of the (re)assignment process.

**Endogenous Participation.** As discussed in Section 4.1, when teachers follow their dominant strategies in the (re)assignment market, their participation decisions remain unaffected by changes in the mechanisms, provided these mechanisms are individually rational and strategy-proof (see Footnote 19). However, if teachers deviate from their dominant strategies, participation decisions could be influenced by a change in the mechanism. Indeed, as observed in our empirical analysis, some teachers may not apply to a school they deem unlikely to be assigned to.

In addition, a change in mechanism may alter the value assigned to a school, as the likelihood of it being exchanged with another school in the future could increase or decrease. In such instances, a change in mechanism, which in turn alters the likelihood of acceptance at various schools and the possibility that a school will be "traded" in the future, could impact participation decisions.

Our SI-CC* simulations provide our empirical counterfactual analysis with more external validity by accounting for some of these participation effects. This is because we found that mobility outcomes for tenured teachers are similar under both SI-CC* and the Current French mechanism, suggesting that participation decisions could be similar under both mechanisms. Moreover, this is achieved without embedding any dynamic incentives in SI-CC, but rather through a simple change in how new teacher preference submissions are handled. More importantly, SI-CC* does lead to a reduction in inequality compared to the status quo, unlike the Current French mechanism. Therefore, it can serve as a good compromise mechanism without employing any of the currently used dynamic incentive features of the Current French mechanism.[65] The optimal design of such incentives in a fully-fledged dynamic model is an interesting future research question.

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initial assignment (See the Ministry website at https://www.education.gouv.fr/mutation-des-personnels-enseignants-du-premier-degre-5498). This "second stage" with position swaps is not allowed for secondary school teachers, as studied in our empirical section.

[65] The Current French mechanism has priorities mostly based on experience level and upgrades for teachers who have worked five or more years at a priority education school. Such schools are in the youngest regions with higher density. This second incentive has worked as a vehicle that channels new teachers to the youngest regions initially and moves them out of these regions to high-experience regions after they gain experience, possibly exacerbating the inequality problem under the current mechanism.

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# Supplemental Appendices

# A Omitted Proofs

## A.1 Proof of Theorem 1

**Proof of Theorem 1.** Consider an arbitrary market $P$. Fix a profile of type rankings $\trianglerighteq$.

**SI-constrained efficiency:** Recall that the requirement of status-quo improvement under $\trianglerighteq$ is embedded in the definition of SI-constrained efficiency. Let $\hat{\mu}$ be the outcome of SI-CC in this market induced by $\trianglerighteq$ for a tiebreaker $\vdash$. We proceed in two parts.

We first show that $\hat{\mu}$ is status-quo improving.

First, consider teachers. Under SI-CC, each school $s$ points to all teachers in $\omega_s$ one by one. When a teacher $t \in \omega_s$ is pointed to by $s$ in some Step $k$, then $s \in A_t^k$ and she can always form a one-school cycle $(s, t)$ whenever she points to $s$. Similarly, any new teacher $t \in N$ can form a cycle with $\emptyset$ in any step of SI-CC. Hence, $\hat{\mu}_t \; R_t \; \omega_t$ for each $t \in T$.

Next, consider schools. The first teacher of any executed chain is a new teacher. Hence, if in some step of SI-CC, a school $s$ is sending out a teacher, then it is simultaneously acquiring another teacher; as a result, $|\hat{\mu}_s| \geq |\omega_s|$. In any Step $k$ of SI-CC, when we consider the set of remaining status-quo employees and teachers assigned in the first $k-1$ steps, because of the positive balance requirement in School Improvement Conditions 1 and 2 and the previous observation, for each school $s$ we achieve Lorenz domination over $\omega_s$. Hence, $\hat{\mu}_s$ is an unambiguous weak improvement over $\omega_s$ for each $s \in S$. This completes the proof that $\hat{\mu}$ is status-quo improving.

Before proving $\hat{\mu}$ cannot be Pareto dominated by another status-quo improving matching for teachers, we first state a claim that will be used in the proof.

*Claim 1.* For a school $s$, suppose Step $K$ is a step in which $s$ is assigned through a chain with $s$ as the last school. Let the set of remaining status-quo employees of $s$ at the end of Step $K$ be denoted by $\omega_s^K$, i.e., $\omega_s^K = \omega_s \cap T^{K+1}$. Let $\gamma$ be a matching such that $\gamma_t = \mu_t^K$ for each $t \in T \setminus T^{K+1}$, i.e., all teachers assigned in the first $K$ steps of SI-CC are matched with their assignment under SI-CC, and $|\gamma_s| < |\omega_s^K \cup \mu_s^K|$. Then, $\gamma$ is not a status-quo improving matching.

*Proof.* Suppose teacher $t \in T^K$ is assigned to $s$ in this chain in Step $K$. By the construction of SI-CC, it must be that $t$ points to $s$ under Condition 2. First observe that, by construction, $\mu_s^K \subseteq \gamma_s$. Also notice that, if $\omega_s^K = \emptyset$, then $|\gamma_s| \geq |\omega_s^K \cup \mu_s^K|$, a contradiction with the assumption of the claim. Hence, $\omega_s^K = T^{K+1} \cap \omega_s \neq \emptyset$. Since school $s$ is the last school in the implemented chain in Step $K$, $\omega_s^K = T^{K+1} \cap \omega_s \subseteq T^K \cap \omega_s \neq \emptyset$. Let $t^s \in T^K \cap \omega_s$ be the teacher pointed by $s$ in Step $K$. Since, by the definition of SI-CC, school improvement condition 1 does not hold for $s$ via teacher $t$ and school improvement condition 2 holds, there exists some type $\theta \neq \theta_\emptyset$ such that $\tau(t^s) \trianglerighteq_s \theta \triangleright_s \tau(t), \theta \triangleright_s \theta_\emptyset$ and[66]

$$\sum_{\theta' \trianglerighteq_s \theta} b_s^{K,\theta'} = \sum_{\theta' \trianglerighteq_s \theta} \left( \left| [\mu_s^{K-1}]^{\theta'} \right| - \left| [\omega_s \setminus T^K]^{\theta'} \right| \right) \leq 0. \tag{1}$$

Since $s$ is the last element of the chain implemented in step $K$, there are two cases to consider:

Case 1: $s$ appears only once in the chain. In this case, we have $\mu_s^K = \mu_s^{K-1} \cup \{t\}$. Since $\theta \triangleright_s \tau(t)$, $\mu_s^K = \mu_s^{K-1} \cup \{t\}$ implies that

$$\sum_{\theta' \trianglerighteq_s \theta} \left| [\mu_s^K]^{\theta'} \right| = \sum_{\theta' \trianglerighteq_s \theta} \left| [\mu_s^{K-1}]^{\theta'} \right|.$$

Case 2: $s$ appears more than once in the chain. Since $s$ can only point to a single tenured teacher, the chain must be of the form $(t_0, s_1, t_1, ..., t_\ell, s, t^s, ..., t, s)$. Hence, $\mu_s^K = \mu_s^{K-1} \cup \{t_\ell, t\} \setminus \{t^s\}$. We note that $\tau(t_\ell) \trianglerighteq_s \theta$. Indeed, $t_\ell$ points to $s$ because improvement condition 1 holds for $s$ via $t_\ell$ and so, given Equation 1, we must have $\tau(t_\ell) \trianglerighteq_s \theta$. Now, since $\tau(t^s) \trianglerighteq_s \theta \triangleright_s \tau(t)$, we have that at Step $K$ one teacher (namely, $t^s$) of type greater than $\theta$ leaves $s$ while two teachers, one with type greater than $\theta$ (namely, $t_\ell$) and one with type worse than $\theta$ join school $s$. From this, we must have

$$\sum_{\theta' \trianglerighteq_s \theta} \left| [\mu_s^K]^{\theta'} \right| = \sum_{\theta' \trianglerighteq_s \theta} \left| [\mu_s^{K-1}]^{\theta'} \right|.$$

Hence we get the above equality in both cases. Now, using Equation 1, we have

$$\sum_{\theta' \trianglerighteq_s \theta} \left| [\mu_s^K]^{\theta'} \right| \leq \sum_{\theta' \trianglerighteq_s \theta} \left| [\omega_s \setminus T^K]^{\theta'} \right|. \tag{2}$$

[66] Actually, the sum of balances $\sum_{\theta' \trianglerighteq_s \theta} b_s^{K,\theta'}$ never becomes negative in the mechanism for any type $\theta$, as the sum starts at zero at the beginning of Step 1, and whenever it is zero, we do not admit a teacher with a type worse than $\theta$ by sending out a teacher with a type better than $\theta$ by Improvement Condition 1.

Given $\mu_s^K \cap \omega_s^K = \emptyset$, we obtain

$$\begin{aligned} \sum_{\theta' \trianglerighteq \theta} \left| \left[\mu_s^K \cup \omega_s^K\right]^{\theta'} \right| &= \sum_{\theta' \trianglerighteq_s \theta} \left| \left[\mu_s^K\right]^{\theta'} \right| + \sum_{\theta' \trianglerighteq_s \theta} \left| \left[\omega_s^K\right]^{\theta'} \right| \\ &\leq \sum_{\theta' \trianglerighteq_s \theta} \left| [\omega_s \setminus T^K]^{\theta'} \right| + \sum_{\theta' \trianglerighteq_s \theta} \left| \left[\omega_s^K\right]^{\theta'} \right| \\ &= \sum_{\theta' \trianglerighteq_s \theta} \left| [\omega_s \setminus T^K]^{\theta'} \right| + \sum_{\theta' \trianglerighteq_s \theta} \left| \left[\omega_s \cap T^{K+1}\right]^{\theta'} \right| \\ &\leq \sum_{\theta' \trianglerighteq_s \theta} \left| [\omega_s \setminus T^K]^{\theta'} \right| + \sum_{\theta' \trianglerighteq_s \theta} \left| \left[\omega_s \cap T^K\right]^{\theta'} \right| \\ &= \sum_{\theta' \trianglerighteq_s \theta} \left| \omega_s^{\theta'} \right| \end{aligned} \tag{3}$$

where the first inequality uses Equation 2 and the penultimate uses the fact that $T^{K+1} \subseteq T^K$.

Finally, $|\gamma_s| < |\omega_s^K \cup \mu_s^K|$—which we assumed—can be written as

$$\sum_{\theta \triangleright_s \theta'} \left| \gamma_s^{\theta'} \right| + \sum_{\theta' \trianglerighteq_s \theta} \left| \gamma_s^{\theta'} \right| < \sum_{\theta \triangleright_s \theta'} \left| \left[\omega_s^K \cup \mu_s^K\right]^{\theta'} \right| + \sum_{\theta' \trianglerighteq_s \theta} \left[\omega_s^K \cup \mu_s^K\right]^{\theta'}.$$

From this, we obtain

$$\begin{aligned} \sum_{\theta' \trianglerighteq_s \theta} \left| \gamma_s^{\theta'} \right| &< \sum_{\theta \triangleright_s \theta'} \left| \left[\omega_s^K \cup \mu_s^K\right]^{\theta'} \right| - \sum_{\theta \triangleright_s \theta'} \left| \gamma_s^{\theta'} \right| + \sum_{\theta' \trianglerighteq_s \theta} \left[\omega_s^K \cup \mu_s^K\right]^{\theta'} \\ &= \sum_{\theta \triangleright_s \theta'} \left| \left[\omega_s^K\right]^{\theta'} \right| + \sum_{\theta \triangleright_s \theta'} \left| \left[\mu_s^K\right]^{\theta'} \right| - \sum_{\theta \triangleright_s \theta'} \left| \gamma_s^{\theta'} \right| + \sum_{\theta' \trianglerighteq_s \theta} \left[\omega_s^K \cup \mu_s^K\right]^{\theta'} \\ &\leq \sum_{\theta \triangleright_s \theta'} \left| \left[\omega_s^K\right]^{\theta'} \right| + \sum_{\theta' \trianglerighteq_s \theta} \left[\omega_s^K \cup \mu_s^K\right]^{\theta'} \\ &= \sum_{\theta' \trianglerighteq_s \theta} \left[\omega_s^K \cup \mu_s^K\right]^{\theta'} \end{aligned} \tag{4}$$

where the second inequality comes from the fact that $\mu_s^K \subseteq \gamma_s$ and the last equality by the fact that all teachers in $\omega_s^K$ have a weakly better type than $\theta$ (since $t^s$ must be the tenured teacher at school $s$ with the lowest type ranking according to $\trianglerighteq_s$). Combining Equations 3 and 4, we obtain

$$\sum_{\theta' \trianglerighteq_s \theta} \left| \gamma_s^{\theta'} \right| < \sum_{\theta' \trianglerighteq_s \theta} \left| \omega_s^{\theta'} \right|.$$

Therefore, $\gamma_s$ is not unambiguous weak improvement over $\omega_s$ for school $s$. $\diamond$

Next, we show that $\hat{\mu}$ cannot be Pareto dominated by another status-quo improving matching for teachers.

On the contrary, suppose there exists a status-quo improving matching $\gamma$ that Pareto dominates $\hat{\mu}$ for teachers. By considering the teachers assigned in each step of SI-CC inductively, we show that such a matching cannot exist, in particular we should have $\gamma = \hat{\mu}$.

We denote the set of teachers assigned in Step $k$ of SI-CC in market $P$ by $I^k$, i.e., $I^k = T^k \setminus T^{k+1}$, and the union of these sets up to Step $k$ as $\overline{I}^k = \cup_{k'=1}^{k} I^{k'}$.

Step 1: Each teacher $t \in I^1$ is assigned in $\hat{\mu}$ to the best school in $A_t^1$. If $\gamma_t \; P_t \; \hat{\mu}_t$ for some $t \in I^1$, then $\gamma_t \notin A_t^1$. Thus, for school $s = \gamma_t$ both school improvement conditions do not hold for teacher $t$. Since this is Step 1, $\mu^0 = \emptyset$, and hence, the current balances $b_s^{0,\theta} = 0$ for each schools $s$ and type $\theta \in \Theta \cup \{\theta_\emptyset\}$. We consider the implications of the violation of the two improvement conditions separately:

(a.) We first show that, since school improvement condition 1 does not hold, $|\gamma_s| > |\omega_s|$. To show this, we consider the following two cases:

(a.1) *There exists a teacher $t^s \in \omega_s$ to whom $s$ is pointing*:

Teacher $t^s$ has type $\tau(t^s) \rhd_s \tau(t)$: thus, $t$ has a worse type than the worst type status-quo employees of this school; or

(a.2) *There does not exist a teacher to whom $s$ is pointing*:

Then, $\omega_s = \emptyset$.

Since $\gamma$ is a status-quo improvement over $\omega_s$, by the first condition of the definition, $|\gamma_s| \geq |\omega_s|$.

Suppose Case (a.1) holds: If $|\gamma_s| = |\omega_s|$, then

$$\sum_{\theta' \trianglerighteq_s \tau(t^s)} |\omega_s^{\theta'}| > \sum_{\theta' \trianglerighteq_s \tau(t^s)} |\gamma_s^{\theta'}|,$$

i.e., $t$ being in $\gamma_s$ violates the Lorenz dominance relation for type $\tau(t^s)$, contradicting $\gamma$ is status-quo improving for $s$. This follows from the fact that $\tau(t^s)$ is the worst type of the status-quo teachers and $\tau(t^s) \rhd_s \tau(t)$. Thus, $|\gamma_s| > |\omega_s|$.

Suppose Case (a.2) holds: Since $t \in \gamma_s$ and $\omega_s = \emptyset$, we have $|\gamma_s| > |\omega_s|$.

Thus, in either case, we have $|\gamma_s| > |\omega_s|$.

(b.) Now, we show that the violation of the school improvement condition 2 for $s$ via $t$ together with $|\gamma_s| > |\omega_s|$ (which we just showed in a.) yields a contradiction. Indeed, the violation of the school improvement condition 2 for $s$ via $t$ implies at least one of the following conditions to hold:

(b.1) $|\omega_s| = q_s$:

In this case, we showed above $|\gamma_s| > |\omega_s|$, which implies $|\gamma_s| > q_s$ contradicting the feasibility of $\gamma$ as matching; or

(b.2) *there are no new teachers*:

In this case, since $|\gamma_s| > |\omega_s|$, there exists some school $s'$ such that $|\omega_{s'}| > |\gamma_{s'}|$, contradicting $\gamma$ status-quo improves $s'$ by the first condition of the status-quo improvement definition; or

(b.3) $\theta_\emptyset \rhd_s \tau(t)$:

Two subcases exist:

- $|\omega_s| = q_s$ or [$\omega_s \neq \emptyset$ and $\theta_\emptyset \rhd_s \tau(t^s)$ where $t^s$ is the teacher pointed by $s$ in Step 1]: As $\omega_s \neq \emptyset$, Case (a.2) does not hold. Thus, Case (a.1) holds and by Case (a.1), $\tau(t^s) \rhd_s \tau(t)$ implying

$$\sum_{\theta' \trianglerighteq_s \tau(t^s)} |\omega_s^{\theta'}| = q_s > \sum_{\theta' \trianglerighteq_s \tau(t^s)} |\gamma_s^{\theta'}|,$$

i.e., $t$ being in $\gamma_s$ violates the Lorenz dominance relation for the worst type status-quo employees of $s$; or

- $\omega_s = \emptyset$ or [$\omega_s \neq \emptyset$ and $\tau(t^s) \rhd_s \theta_\emptyset$ where $t^s$ is the teacher pointed by $s$ in Step 1]: For the vacant seat type $\theta_\emptyset$,

$$\sum_{\theta' \trianglerighteq_s \theta_\emptyset} |\omega_s^{\theta'}| = q_s > \sum_{\theta' \trianglerighteq_s \theta_\emptyset} |\gamma_s^{\theta'}|,$$

i.e., $t$ being in $\gamma_s$ violates the Lorenz dominance relation for $\theta_\emptyset$.

Either subcase contradicts $\gamma$ status-quo improves $s$.

Then, school improvement conditions 1 and 2 cannot be violated, and $t$ could have been pointed to $s$ in Step 1, which is a contradiction.

Hence, such a teacher $t$ cannot exist with $\gamma_t P_t \hat{\mu}_t$. Since $\gamma_t \, R_t \, \hat{\mu}_t$ for each $t$ then for each $t \in I^1$, $\gamma_t = \hat{\mu}_t$.

*Inductive assumption:* For any $k > 1$, assume that for each $k' < k$ and $t \in I^{k'}$, $\gamma_t = \hat{\mu}_t$. We show that the same holds for teachers in $I^k$:

*Step k:* Each teacher $t \in I^k$ is assigned in $\hat{\mu}$ to the best school in $A_t^k$. If $\gamma_t \, P_t \, \hat{\mu}_t$ for some $t \in I^k$, then $\gamma_t \notin A_t^k$. Thus, for school $s = \gamma_t$ both school improvement conditions are violated via teacher $t$. We consider the implications of the violation of the two improvement conditions separately:

(a.) We first show that the violation of school improvement condition 1 yields

$$|\gamma_s \setminus \bar{I}^{k-1}| > |\omega_s \setminus \bar{I}^{k-1}|. \tag{5}$$

In order to do so, we consider the following two cases:

(a.1) *There exists a teacher $t^s \in \omega_s \cap T^k$ to whom $s$ is pointing*:

Then it has type $\tau(t^s) \rhd_s \tau(t)$ and there exists an intermediate type $\theta$ such that $\tau(t^s) \unrhd_s \theta \rhd_s \tau(t)$ with

$$0 \geq \begin{cases} \sum_{\theta' \unrhd_s \theta} \left( \left| [\mu_s^{k-1}]^{\theta'} \right| - \left| [\omega_s \setminus T^k]^{\theta'} \right| \right) & \text{if } \theta \rhd_s \theta_\emptyset \\ \sum_{\theta' \unrhd_s \theta, \theta' \neq \theta_\emptyset} \left( \left| [\mu_s^{k-1}]^{\theta'} \right| - \left| [\omega_s \setminus T^k]^{\theta'} \right| \right) + \underbrace{\left( |\omega_s \setminus T^k| - |\mu_s^{k-1}| \right)}_{= b_s^{k,\theta_\emptyset} = -\#\text{ filled vacant seats}} & \text{if } \theta_\emptyset \unrhd_s \theta \end{cases} \tag{6}$$

By the inductive assumption, $\mu_{t'}^{k-1} = \gamma_{t'}$ for each $t'$ assigned until this step (i.e., those in $\overline{I}^{k-1}$), and hence we can rewrite Inequality 6 as follows,

$$0 \geq \begin{cases} \sum_{\theta' \unrhd_s \theta} \left( \left| [\gamma_s \cap \overline{I}^{k-1}]^{\theta'} \right| - \left| [\omega_s \cap \overline{I}^{k-1}]^{\theta'} \right| \right) & \text{if } \theta \rhd_s \theta_\emptyset \\ \sum_{\theta' \unrhd_s \theta, \theta' \neq \theta_\emptyset} \left( \left| [\gamma_s \cap \overline{I}^{k-1}]^{\theta'} \right| - \left| [\omega_s \cap \overline{I}^{k-1}]^{\theta'} \right| \right) + \left( |\omega_s \cap \overline{I}^{k-1}| - |\gamma_s \cap \overline{I}^{k-1}| \right) & \text{if } \theta_\emptyset \unrhd_s \theta \end{cases} \tag{7}$$

By the definition of SI-CC, teacher $t$ has a worse type than the remaining worst-type status-quo employee of this school, i.e., those in $\omega_s \setminus \overline{I}^{k-1}$.

We first consider the case $\theta \rhd_s \theta_\emptyset$. Suppose $|\gamma_s \setminus \overline{I}^{k-1}| \leq |\omega_s \setminus \overline{I}^{k-1}|$, then as $\tau(t^s) \unrhd_s \theta$ for the worst-type remaining employee $t^s \in \omega_s \setminus \overline{I}^{k-1}$ and the fact that only status-quo teachers of school $s$ who have weakly worse types than $\tau(t^s)$ have been assigned in previous steps of SI-CC, we have

$$|\omega_s \setminus \overline{I}^{k-1}| = \sum_{\theta' \unrhd_s \theta} \left| [\omega_s \setminus \overline{I}^{k-1}]^{\theta'} \right| = \sum_{\theta' \unrhd_s \theta} \left( \left| \omega_s^{\theta'} \right| - \left| [\omega_s \cap \overline{I}^{k-1}]^{\theta'} \right| \right). \tag{8}$$

On the other hand, teachers with worse type than $\theta$ are in $\gamma_s \setminus \overline{I}^{k-1}$, e.g, $t \in \gamma_s \setminus \overline{I}^{k-1}$, and hence,

$$|\gamma_s \setminus \overline{I}^{k-1}| > \sum_{\theta' \unrhd_s \theta} \left| [\gamma_s \setminus \overline{I}^{k-1}]^{\theta'} \right| = \sum_{\theta' \unrhd_s \theta} \left( \left| \gamma_s^{\theta'} \right| - \left| [\gamma_s \cap \overline{I}^{k-1}]^{\theta'} \right| \right). \tag{9}$$

Then the supposition that $|\gamma_s \setminus \overline{I}^{k-1}| \leq |\omega_s \setminus \overline{I}^{k-1}|$, first part of Equation 7, and Equations 8 and 9 together imply

$$\sum_{\theta' \unrhd_s \theta} \left( \left| \gamma_s^{\theta'} \right| - \left| \omega_s^{\theta'} \right| \right) < 0,$$

further implying

$$\sum_{\theta' \unrhd_s \theta} |\omega_s^{\theta'}| > \sum_{\theta' \unrhd_s \theta} |\gamma_s^{\theta'}|,$$

i.e., $\gamma$ is not a status-quo improvement, a contradiction. Thus, when $\theta \rhd_s \theta_\emptyset$, we should also have

$$|\gamma_s \setminus \overline{I}^{k-1}| > |\omega_s \setminus \overline{I}^{k-1}|. \tag{10}$$

Next, we consider the case $\theta_\emptyset \trianglerighteq_s \theta$. Equations 8 and 9 still hold, excluding from the sum $\theta' = \theta_\emptyset$. That is,

$$|\omega_s \setminus \overline{I}^{k-1}| = \sum_{\theta' \trianglerighteq_s \theta,\, \theta' \neq \theta_\emptyset} \left( \left|\omega_s^{\theta'}\right| - \left|[\omega_s \cap \overline{I}^{k-1}]^{\theta'}\right| \right). \tag{11}$$

$$|\gamma_s \setminus \overline{I}^{k-1}| > \sum_{\theta' \trianglerighteq_s \theta,\, \theta' \neq \theta_\emptyset} \left( \left|\gamma_s^{\theta'}\right| - \left|[\gamma_s \cap \overline{I}^{k-1}]^{\theta'}\right| \right). \tag{12}$$

The facts that $|\omega_s \setminus \overline{I}^{k-1}| = |\omega_s| - |\omega_s \cap \overline{I}^{k-1}|$ and $|\gamma_s \setminus \overline{I}^{k-1}| = |\gamma_s| - |\gamma_s \cap \overline{I}^{k-1}|$ together with the second part of Equation 7 imply

$$0 \geq \sum_{\theta' \trianglerighteq_s \theta,\, \theta' \neq \theta_\emptyset} \left( \left|[\gamma_s \cap \overline{I}^{k-1}]^{\theta'}\right| - \left|[\omega_s \cap \overline{I}^{k-1}]^{\theta'}\right| \right) - |\omega_s \setminus \overline{I}^{k-1}| + |\gamma_s \setminus \overline{I}^{k-1}| + |\omega_s| - |\gamma_s| \tag{13}$$

Equations 11, 12, and 13 imply

$$\sum_{\theta' \trianglerighteq_s \theta,\, \theta' \neq \theta_\emptyset} \left( \left|\gamma_s^{\theta'}\right| - \left|\omega_s^{\theta'}\right| \right) + \left( |\omega_s| - |\gamma_s| \right) < 0.$$

Hence, we have

$$\sum_{\theta' \trianglerighteq_s \theta} |\omega_s^{\theta'}| > \sum_{\theta' \trianglerighteq_s \theta} |\gamma_s^{\theta'}|,$$

i.e., when $\theta_\emptyset \trianglerighteq_s \theta$, $\gamma$ is not a status-quo improvement, a contradiction. Thus, this case (i.e., $\theta_\emptyset \trianglerighteq_s \theta$) cannot hold. Therefore, we have $\theta \rhd_s \theta_\emptyset$ and Equation 5 holds in order $\gamma$ to be a status-quo improvement.

(a.2) *School $s$ is not pointing to any teacher:*

Then, $\omega_s \setminus \overline{I}^{k-1} = \emptyset$, and hence, as $t \in \gamma_s \setminus \overline{I}^{k-1}$ we have $|\gamma_s \setminus \overline{I}^{k-1}| > |\omega_s \setminus \overline{I}^{k-1}|$.

Observe that in the algorithm of SI-CC at each step we make sure that each school acquires at least as many teachers as it sends out and hence, $|\mu_s^{k-1}| \geq |\omega_s \cap \overline{I}^{k-1}|$. Since $\mu_s^{k-1} = \gamma_s \cap \overline{I}^{k-1}$ by the inductive assumption, we have $|\gamma_s \cap \overline{I}^{k-1}| \geq |\omega_s \cap \overline{I}^{k-1}|$. Therefore, as we also showed that $|\gamma_s \setminus \overline{I}^{k-1}| > |\omega_s \setminus \overline{I}^{k-1}|$ holds regardless of either Case (a.1) or Case (a.2) holds, we obtain $|\gamma_s| > |\omega_s|$.

(b.) The violation of the school improvement condition 2 for $s$ via $t$, on the other hand, implies at least one of the following three conditions to hold:

(b.1) *All vacant seats of school $s$ have been assigned to teachers in $\overline{I}^{k-1}$, i.e., $|\omega_s \setminus \overline{I}^{k-1}| = q_s - |\mu_s^{k-1}|$:*

By the inductive assumption, $\mu_{t'}^{k-1} = \gamma_{t'}$ for each $t'$ assigned until this step (i.e., those in $\overline{I}^{k-1}$), we have

$$q_s - |\gamma_s \cap \overline{I}^{k-1}| = |\omega_s \setminus \overline{I}^{k-1}|.$$

By the feasibility of a matching, we have

$$q_s - |\gamma_s \cap \overline{I}^{k-1}| \geq |\gamma_s \setminus \overline{I}^{k-1}|.$$

In Case (a), we showed that, for $\gamma$ to be a status-quo improvement, $|\gamma_s \setminus \overline{I}^{k-1}| > |\omega_s \setminus \overline{I}^{k-1}|$. Then, we have

$$|\omega_s \setminus \overline{I}^{k-1}| = q_s - |\gamma_s \cap \overline{I}^{k-1}| \geq |\gamma_s \setminus \overline{I}^{k-1}| > |\omega_s \setminus \overline{I}^{k-1}|,$$

which is a contradiction.

(b.2) *There are no remaining new teachers, i.e., $N \cap T^k = N \setminus \overline{I}^{k-1} = \emptyset$:*

We know from Case (a) that Equation 5 holds, i.e.,

$$\left|\gamma_s \setminus \overline{I}^{k-1}\right| > \left|\omega_s \setminus \overline{I}^{k-1}\right|.$$

If there are no remaining new teachers at Step $k$, then in the future steps of SI-CC, including Step $k$, only the remaining tenured teachers can be assigned in the algorithm. By this, together with $\mu_{t'}^{k-1} = \gamma_{t'}$ for each $t' \in \overline{I}^{k-1}$ (i.e., the inductive assumption), and that $\gamma$ is individually rational[67] , we have

$$\bigcup_{s' \in S} \gamma_{s'} \setminus \overline{I}^{k-1} = \bigcup_{s' \in S} \omega_{s'} \setminus \overline{I}^{k-1}.$$

This implies by Equation 5 that there exists some school $s' \neq s$ such that

$$\left|\gamma_{s'} \setminus \overline{I}^{k-1}\right| < \left|\omega_{s'} \setminus \overline{I}^{k-1}\right|, \tag{14}$$

Given that $\gamma_{s'}$ status-quo improves $s'$, and hence $|\gamma_{s'}| \geq |\omega_{s'}|$, we further have

$$\left|\gamma_{s'} \cap \overline{I}^{k-1}\right| = |\gamma_{s'}| - \left|\gamma_{s'} \setminus \overline{I}^{k-1}\right| > |\omega_{s'}| - \left|\omega_{s'} \setminus \overline{I}^{k-1}\right| = \left|\omega_{s'} \cap \overline{I}^{k-1}\right|.$$

Since $\mu_{t'}^{k-1} = \gamma_{t'}$ for each $t' \in \overline{I}^{k-1}$ by the inductive assumption,

$$\left|\mu_{s'}^{k-1}\right| = \left|\gamma_{s'} \cap \overline{I}^{k-1}\right| > \left|\omega_{s'} \cap \overline{I}^{k-1}\right|. \tag{15}$$

[67] Recall that $\omega_t \ R_t \ \emptyset$ for all $t \in T$.

Thus, a chain was implemented in a previous step in the SI-CC algorithm, and $s'$ was the last school in this chain. Let $\ell \leq k-1$ be the last step at which such a chain was implemented. Equation 14 imply

$$|\gamma_{s'}| = \left|\gamma_{s'} \backslash \bar{I}^{k-1}\right| + \left|\mu_{s'}^{k-1}\right| < \left|\omega_{s'} \backslash \bar{I}^{k-1}\right| + \left|\mu_{s'}^{k-1}\right| = \left|(\omega_{s'} \backslash \bar{I}^{k-1}) \cup \mu_{s'}^{k-1}\right| \quad (16)$$

where the first equality follows from the first equality in Equation 15 (which follows from the inductive assumption) and the last equality follows from the fact that $\mu_{s'}^{k-1} \subseteq \bar{I}^{k-1}$.

Note that, after replacing $k-1$ with $k'$ in every place, Equation 16 holds for each step $k'$ with $\ell < k' \leq k-1$. To see this, recall that by the definition of Step $\ell$, either a cycle or a chain in which $s'$ was not the last school was implemented at any step following Step $\ell$. In both cases, if $s'$ was part of this cycle or chain, the number of status-quo teachers leaving equals the number of teachers entering so that the Inequality 14 written for Step $k'$ instead of $k-1$ remains correct, implying Inequality 16 also holds for Step $k'$.

So at the end of Step $\ell$, a chain was just implemented where $s'$ was the last school of the chain and $|\gamma_{s'}| < \left|(\omega_{s'} \backslash \bar{I}^{\ell-1}) \cup \mu_{s'}^{\ell-1}\right|$. But then, by Claim 1 applied for Step $\ell$ and school $s'$, we conclude that $\gamma$ is not status-quo improving, which is a contradiction.

(b.3) $\sum_{\theta' \trianglerighteq_s \theta} b_s^{k,\theta'} \leq 0$ *for some* $\theta$ *such that* $\theta_\emptyset \trianglerighteq_s \theta \rhd_s \tau(t)$:

First, notice that the explanation provided in Case (a.1) when $\theta_\emptyset \trianglerighteq_s \theta$ holds even if $\omega_s \cap T^k = \emptyset$. Hence, in this case, by following the exact steps as in Case (a.1), we show including $t$ in $\gamma$ violates status-quo improvement for $s$, a contradiction.

Then, Condition 2 cannot be violated as none of these conditions hold, consequently implying that $t$ can point to school $s$ in Step $k$ of the SI-CC algorithm, which is a contradiction. Hence, such a teacher $t \in I^k$ with $\gamma_t \, P_t \, \hat{\mu}_t$ cannot exist.

Since $\gamma_t \, R_t \, \hat{\mu}_t$ for each $t$, we have for each $t \in I^k$, $\gamma_t = \hat{\mu}_t$, completing the induction and showing that $\gamma = \hat{\mu}$, and hence, $\hat{\mu}$ is constrained efficient.

**Strategy-proofness:** We state two claims that we will use in the proof.

*Claim 2*. Suppose teacher $t$ is assigned in Step $K$ of SI-CC. For any $k < K$, $A_t^{k+1} \subseteq A_t^k$.

*Proof.* Let $s \notin A_t^k$. We show that $s \notin A_t^{k+1}$. We consider two possible cases.

*Case 1. School $s$ does not have a vacant position at Step $k$:* First, notice that, if there is no remaining status-quo employee of $s$ in Step $k$, then $s$ should have been removed in an earlier step of SI-CC. Thus, there exists a remaining status-quo teacher of $s$ in Step $k$. Then, $s$ points to a teacher in $\omega_s \cap T^k$, say $t^s$ in Step $k$. As $s \notin A_t^k$, there exists some type $\theta$ such that $\tau(t^s) \trianglerighteq_s \theta \rhd_s \tau(t)$ with

$$\sum_{\theta' \trianglerighteq_s \theta} b_s^{k,\theta'} \leq 0. \tag{17}$$

Also, notice that $s$ cannot be the last school in an executed chain in Step $k$. [68] That is, $s$ cannot get a teacher without sending out a status-quo employee. Moreover, $s$ cannot send out a status-quo employee without getting a new one by the definition of SI-CC.

If school $s$ is part of the executed cycle or chain in Step $k$, then the teacher assigned to $s$ has a type weakly better than type $\theta$ under $\rhd_s$ and similarly, the teacher leaving school $s$, namely, $t^s$ also has a type weakly better than type $\theta$. Hence, after executing the cycle or chain in Step $k$, Inequality 17 still holds after replacing $k$ with $k+1$. If $\omega_s \cap T^{k+1} = \emptyset$, then $s$ is removed at the end of Step $k$. Otherwise, $s$ points to a teacher in Step $k+1$ who has a type weakly better than $\tau(t^s)$ under $\trianglerighteq_s$.

If school $s$ is not part of the executed cycle or chain in Step $k$, Inequality 17 still holds after replacing $k$ with $k+1$ as $s$'s current match and its set of remaining status-quo teachers does not change in the algorithm.

In either case, $s \notin A_t^{k+1}$.

*Case 2. School s has a vacant position at Step k:* Either (i) $\sum_{\theta' \trianglerighteq_s \theta} b_s^{k,\theta'} \leq 0$ for some type $\theta$ such that $\theta_\emptyset \trianglerighteq_s \theta \rhd_s \tau(t)$ or (ii) $\sum_{\theta' \trianglerighteq_s \theta} b_s^{k,\theta'} > 0$ for every $\theta$ such that $\theta_\emptyset \trianglerighteq_s \theta \rhd_s \tau(t)$ but there does not exist a remaining new teacher in Step $k$. By the definition of SI-CC, under the latter case, a cycle needs to be executed in Step $k$.

First, notice that if school $s$ is not part of the executed cycle or chain in Step $k$, then both cases (i) and (ii) continue to hold for Step $k+1$, and therefore, $s \notin A_t^{k+1}$.

Second, suppose a chain is executed in Step $k$ and $s$ is a part of that chain. Then, as explained above, case (ii) cannot hold and thus case (i) holds. Without loss of generality, suppose $\theta$ is the lowest ranked type under $\rhd_s$ such that case (i) holds.

If $s$ is the last school of the executed chain, then the teacher assigned to $s$, say $t'$, has a type weakly better than type $\theta$ and occupies a vacant seat. Since $\theta_\emptyset \rhd_s \tau(t)$ and $\tau(t') \rhd_s \tau(t)$, case (i) still holds in Step $k+1$, and therefore $s \notin A_t^{k+1}$.

If $s$ is not the last school in the executed chain, and $t'$ is assigned to $s$. Then, $s$ points to a teacher in $\omega_s \cap T^k$, say $t^s$, as explained in Case 1 above, and $\tau(t^s) \rhd \tau(t)$. Since both $t'$ and $t^s$ have types weakly better than $\theta$, after executing the chain in Step $k$, case (i) still holds for Step $k+1$, and therefore $s \notin A_t^{k+1}$.

Third, suppose, instead of a chain, a cycle is executed in Step $k$, and $s$ is in this cycle. By following Case 1's reasoning, we show that $s \notin A_t^{k+1}$. $\diamond$

[68] Otherwise, improvement condition 2 for $s$ would be satisfied but then $s$ would have available seats.

*Claim 3*. Consider a Step $k$ of SI-CC mechanism such that there exists a path of schools and teachers $(s_1, t_1, s_2, t_2, \ldots, s_\ell, t_\ell)$ in which for each $\ell' \leq \ell$, school $s_{\ell'}$ points to teacher $t_{\ell'}$ and teacher $t_{\ell'-1}$ points, according to the school improvement condition 1, to school $s_{\ell'}$ for each $\ell' \leq \ell$ and $s_1 \in A^k_{t_\ell}$. If none of the schools in this path are assigned a teacher in this step, the same path forms in Step $k+1$ and $s_1 \in A^{k+1}_{t_\ell}$.

*Proof.* As no teacher is assigned to the schools of the path in Step $k$, the teachers in the path remain in Step $k+1$. Since $t_{\ell'} = t^{s_{\ell'}}$ is the highest priority remaining status-quo employee under the pointing order in Step $k$ of school $s_{\ell'}$, she continues to be so in Step $k+1$, thus, school $s_{\ell'}$ points to $t_{\ell'}$ in Step $k+1$. Moreover, no other status-quo employee of these schools is assigned to any other school in Step $k$, because the assignment of status-quo employees requires the school pointing to them and each school points to at most one teacher in this step. Thus, as school improvement condition 1 holds for each school $s_{\ell'}$ via teacher $t_{\ell'-1}$ in Step $k$, the same condition continues to hold in Step $k+1$ via the same teacher. Hence, $s_{\ell'} \in A^{k+1}_{t_{\ell'-1}}$ for each $\ell'$. Since $A^{k+1}_{t_{\ell'-1}} \subseteq A^k_{t_{\ell'-1}}$ by Claim 2, and $s_{\ell'}$ is the favorite school of teacher $t_{\ell'-1}$ in the opportunity set in Step $k$, we still have $s_{\ell'}$ as the favorite school of teacher $t_{\ell'-1}$ in Step $k+1$ and she continues to point to $s_{\ell'}$ in Step $k+1$. $\diamond$

We are ready to finish the proof for the strategy-proofness of SI-CC. Recall that we denote the set of teachers assigned in Step $k$ of SI-CC by $I^k$.

Notice that any teacher $t$ cannot affect $A^1_t$ by misreporting her preferences since $A^1_t$ does not depend on the submitted preferences. Moreover, by Claim 2, $\{A^{k'}_t\}_{k'}$, the opportunity sets for teacher $t$, weakly shrink throughout SI-CC. Hence, any teacher $t$ cannot be assigned to a school $s \notin A^1_t$ under SI-CC.

First, we consider the teachers in $I^1$. Each teacher $t \in I^1$ is assigned to her best choice in $A^1_t$. Hence, any teacher $t \in I^1$ cannot benefit from misreporting her preferences.

Next, we consider a teacher $t \in I^2$. As explained above, teacher $t$ cannot be assigned to school $s \notin A^1_t$ under SI-CC. Teacher $t \in I^2$ is assigned to her favorite school in $A^2_t$ when she submits her true preferences. We denote her favorite school in $A^2_t$ according to $P_t$ by $s'$. By Claim 2, $A^2_t \subseteq A^1_t$. Hence, if $t \in I^2$ can benefit from misreporting her preferences, then she is assigned to some school $s \in A^1_t \setminus A^2_t$. We will show that $s \, P_t \, s'$ and $s \in A^1_t \setminus A^2_t$ lead to a contradiction.Particularly, we show $t$ cannot prevent the cycle or chain executed in step 1 without hurting herself.

If $t$ forms a cycle in step 1 by misreporting and pointing to some school $s'' \in A^1_t$, then by Claim 3, $s'' \in A^2_t$ and the path leading to $t$ in this cycle starting with school $s''$ forms again when she submits her true preference relation $P_t$, which does not match her in step 1. Hence, any such school $s''$ cannot be preferred to $s'$, which is $t$'s assignment under truthtelling.

If a chain is executed in step 1 when teacher $t$ is truthful, teacher $t$ cannot be a part of an executed chain by misreporting and pointing to some other school in $A_t^1$. This follows from the fact that the executed chain starts with a specific new teacher and a teacher $\bar{t}$, who is pointed to by her status-quo school $\omega_{\bar{t}}$, can only be added to the executed chain if a previously included teacher points to school $\omega_{\bar{t}}$, independent of $\bar{t}$'s preferences.

Thus, teacher $t$ can prevent the executed chain by only forming a cycle by misreporting. However, as explained above, in such a cycle, $t$ will be assigned to a school weakly worse than $s'$.

Moreover, with a similar reasoning to a chain, teacher $t$ cannot affect the executed cycles in step 1 by submitting a different preference list without being matched in step 1 in a new cycle (and therefore, making her weakly worse off as we showed above).

By using similar arguments, we can show that any teacher in $I^k$ where $k > 2$ cannot benefit from misreporting her preferences.

## A.2 Proof of Proposition 1

Before moving to the proof of Proposition 1, we start by proving Lemma 1.

**Proof of Lemma 1.** Let $\trianglerighteq$ be the type ranking profile induced by the partition $(L, H)$ and $\mu$ be a matching which is status-quo improving for schools under $\trianglerighteq$ when $\omega$ is the status-quo matching.

Fix $s \in L$. By the construction of $\trianglerighteq_s$ and by the definition of the status-quo improvement for schools property, we have that for $k = 0, \ldots, K-2$:

$$\begin{aligned}
&\sum_{\ell=0}^{k} |\mu_s^{\theta_{K-1-\ell}}| \geq \sum_{\ell=0}^{k} |\omega_s^{\theta_{K-1-\ell}}| \\
\iff \quad & \sum_{\ell=0}^{k} |E_s^{\theta_{K-1-\ell}}| + \sum_{\ell=0}^{k} |\mu_s^{\theta_{K-1-\ell}}| \geq \sum_{\ell=0}^{k} |E_s^{\theta_{K-1-\ell}}| + \sum_{\ell=0}^{k} |\omega_s^{\theta_{K-1-\ell}}| \\
\iff \quad & \sum_{\ell=0}^{k} \delta_s^{\mu, v_{K-1-\ell}} \geq \sum_{\ell=0}^{k} \delta_s^{\omega, v_{K-1-\ell}} \qquad \text{after dividing both sides by } |E_s| + q_s.
\end{aligned}$$

For $k = K-1$, by the status-quo improvement for schools property, Lorenz dominance condition at $\theta_\emptyset$ implies

$$\begin{aligned}
&\sum_{\ell=0}^{K-1} |\mu_s^{\theta_{K-1-\ell}}| + (q_s - |\mu_s|) \geq \sum_{\ell=0}^{K-1} |\omega_s^{\theta_{K-1-\ell}}| + (q_s - |\omega_s|) \\
\iff \quad & \sum_{\ell=0}^{K-1} |E_s^{\theta_{K-1-\ell}}| + \sum_{\ell=0}^{K-1} |\mu_s^{\theta_{K-1-\ell}}| + (q_s - |\mu_s|) \geq \sum_{\ell=0}^{K-1} |E_s^{\theta_{K-1-\ell}}| + \sum_{\ell=0}^{k} |\omega_s^{\theta_{K-1-\ell}}| + (q_s - |\omega_s|) \\
\iff \quad & \sum_{\ell=0}^{K-1} \delta_s^{\mu, v_{K-1-\ell}} \geq \sum_{\ell=0}^{K-1} \delta_s^{\omega, v_{K-1-\ell}} \qquad \text{after dividing both sides by } |E_s| + q_s.
\end{aligned}$$

Thus, we deduce that $\delta_s^\mu$ LDs $\delta_s^\omega$. Since $f$ is LD-increasing, we have that $f(\delta_s^\mu) \geq f(\delta_s^\omega)$.

Now, fix $s \in H$. By the definition of the status-quo improvement for schools property, Lorenz dominance condition at $\theta_\emptyset$ implies

$$\begin{aligned}
& |\mu_s^{\theta_0}| + |\mu_s^{\theta_\emptyset}| \geq |\omega_s^{\theta_0}| + |\omega_s^{\theta_\emptyset}| \\
\iff\quad & |\mu_s^{\theta_0}| + (q_s - |\mu_s|) \geq |\omega_s^{\theta_0}| + (q_s - |\omega_s|) \\
\iff\quad & |E_s^{\theta_0}| + |\mu_s^{\theta_0}| + (q_s - |\mu_s|) \geq |E_s^{\theta_0}| + |\omega_s^{\theta_0}| + (q_s - |\omega_s|) \\
\iff\quad & \delta_s^{\mu,0} \geq \delta_s^{\omega,0} \qquad \text{after dividing both sides by } |E_s| + q_s \\
\iff\quad & 1 - \delta_s^{\mu,v_0} \leq 1 - \delta_s^{\omega,v_0} \\
\iff\quad & \sum_{\ell=0}^{K-2} \delta_s^{\mu,v_{K-1-\ell}} \leq \sum_{\ell=0}^{K-2} \delta_s^{\omega,v_{K-1-\ell}}.
\end{aligned}$$

Note also that $\sum_{\ell=0}^{K-1} \delta_s^{\mu,v_{K-1-\ell}} = 1 = \sum_{\ell=0}^{K-1} \delta_s^{\omega,v_{K-1-\ell}}$ so that trivially $\sum_{\ell=0}^{K-1} \delta_s^{\mu,v_{K-1-\ell}} \leq \sum_{\ell=0}^{K-1} \delta_s^{\omega,v_{K-1-\ell}}$. Now, using the Lorenz domination condition of the status-quo improvement for schools property at any $\theta_k$ such that $\theta_\emptyset \rhd_s \theta_k$—that is $k = 1, \ldots, K-1$—we have:

$$\begin{aligned}
& |\mu_s^{\theta_\emptyset}| + \sum_{\ell=0}^{k} |\mu_s^{\theta_\ell}| \geq |\omega_s^{\theta_\emptyset}| + \sum_{\ell=0}^{k} |\omega_s^{\theta_\ell}| \\
\iff\quad & (q_s - |\mu_s|) + \sum_{\ell=0}^{k} |\mu_s^{\theta_\ell}| \geq (q_s - |\omega_s|) + \sum_{\ell=0}^{k} |\omega_s^{\theta_\ell}| \\
\iff\quad & \sum_{\ell=0}^{K-1} |E_s^{\theta_\ell}| + (q_s - |\mu_s|) + \sum_{\ell=0}^{k} |\mu_s^{\theta_\ell}| \geq \sum_{\ell=0}^{K-1} |E_s^{\theta_\ell}| + (q_s - |\omega_s|) + \sum_{\ell=0}^{k} |\omega_s^{\theta_\ell}| \\
\iff\quad & \sum_{\ell=0}^{k} \delta_s^{\mu,v_\ell} \geq \sum_{\ell=0}^{k} \delta_s^{\omega,v_\ell} \qquad \text{after dividing both sides by } |E_s| + q_s \\
\iff\quad & 1 - \sum_{\ell=0}^{k} \delta_s^{\mu,v_\ell} \leq 1 - \sum_{\ell=0}^{k} \delta_s^{\omega,v_\ell} \\
\iff\quad & \sum_{\ell=0}^{K-2-k} \delta_s^{\mu,v_{K-1-\ell}} \leq \sum_{\ell=0}^{K-2-k} \delta_s^{\omega,v_{K-1-\ell}}.
\end{aligned}$$

We conclude that $\delta_s^\omega$ LDs $\delta_s^\mu$. Since $f$ is LD-increasing, we have that $f(\delta_s^\omega) \geq f(\delta_s^\mu)$.

**Proof of Proposition 1.** Fix an inequality index $\mathcal{I}$, a continuous and LD-increasing statistic $f$, and a base generic economy $\langle T, \Theta, \tau, S, q, P, \omega; E^1, \mathcal{V}\rangle$. Let us start by defining, for each $n$-economy $E^n$, the *relevant domain of* $\mathcal{I}$ for the statistic $f$ as

$$\mathcal{Z}^{f,n} = co\left\{f(\delta^{\mu,n}) : \mu \in \mathcal{M}\right\}.$$

In the sequel, we let $\mathcal{O}$ be the set of points at which $\mathcal{I}$ is continuously differentiable (which, by assumption, is an open and dense set of values in $\mathbb{R}^{|S|}$).

Since genericity holds, by definition, we have that $\frac{\partial \mathcal{I}}{\partial z_s}(z^{*f}) \neq 0$ for each school $s$ and $z^{*f} \in \mathcal{O}$. Since $\mathcal{O}$ is an open set, there is $\varepsilon' > 0$ small enough such that $B_{\varepsilon'}(z^{*f}) \subset \mathcal{O}$.[69] Hence, for each $z \in B_{\varepsilon'}(z^{*f})$, $\mathcal{I}$ is continuously differentiable. Since derivatives of $\mathcal{I}$ at $z^{*f}$ are non-zero, by continuity, there must be $\varepsilon \in (0, \varepsilon')$ such that, for each school $s$, either

$$\frac{\partial \mathcal{I}}{\partial z_s}(z) > 0 \text{ for each } z \in B_\varepsilon(z^{*f})$$

or

$$\frac{\partial \mathcal{I}}{\partial z_s}(z) < 0 \text{ for each } z \in B_\varepsilon(z^{*f}).$$

The set of schools for each of which the former inequality is satisfied is $H^{\mathcal{I},f}$ as $z^{*f} \in B_\varepsilon(z^{*f})$ while the set of schools for each of which the latter inequality is satisfied is $L^{\mathcal{I},f}$ for the same reason. Note that, in our sequence of replica economies, for each school $s$, $\delta_s^{\mu,n}$ converges to $\delta_s^*$. Therefore, by continuity of statistic $f$, for $n$ large enough, $f(\delta^{\mu,n}) = \left(f(\delta_s^{\mu,n})\right)_{s\in S} \in B_\varepsilon(z^{*f})$ for each matching $\mu$.[70] Since $B_\varepsilon(z^{*f})$ is convex, this also implies that convex combination of the points $\{f(\delta^{\mu,n}) : \mu \in \mathcal{M}\}$ is in $B_\varepsilon(z^{*f})$. Hence, the relevant domain $\mathcal{Z}^{f,n}$ of $\mathcal{I}$ in an $n$-economy is included in $B_\varepsilon(z^{*f})$. In the sequel, we fix such a large enough $n$.

First, this implies that $\mathcal{I}$ has well-defined partial derivatives everywhere on its relevant domain. Since for each $z$ in this relevant domain, there is an open neighborhood of $z$ in which partial derivatives are well-defined and continuous, this also implies that total derivatives of $\mathcal{I}$ are well-defined on this relevant domain.

Second, this also implies that, for each school $s \in H^{\mathcal{I},f}$, $\frac{\partial \mathcal{I}}{\partial z_s}$ is strictly positive on its relevant domain $\mathcal{Z}^{f,n}$ and for each school $s \in L^{\mathcal{I},f}$, $\frac{\partial \mathcal{I}}{\partial z_s}$ is strictly negative on this relevant domain $\mathcal{Z}^{f,n}$.

With these in mind, let $\mu$ be a status-quo improving matching for schools for the natural type ranking profile of $\mathcal{I}$ and $f$. We define $\Delta_s = f(\delta_s^\mu) - f(\delta_s^\omega)$ as the change in the statistic value from $\omega$ to $\mu$ for each school $s$. Because $\mu$ status-quo improves $\omega$ for schools for natural type ranking profile and $f$ is LD-increasing, we have $\Delta_s \geq 0$ for each $s \in L^{\mathcal{I},f}$ and $\Delta_s \leq 0$ for each $s \in H^{\mathcal{I},f}$ by Lemma 1. Since $\mathcal{I}$ is differentiable in $\mathcal{Z}^{f,n}$, its total differential satisfies $d\mathcal{I} = \sum_{s\in S} \frac{\partial \mathcal{I}}{\partial z_s}(z) dz_s$. Then, to calculate the total variation in the inequality index from $\omega$ to $\mu$, we obtain the following path integral on the linear path in variable

[69] Using standard notations, $B_{\varepsilon'}(z^{*f})$ denotes the open ball with radius $\varepsilon'$ around $z^{*f}$.

[70] $\varepsilon > 0$ can be taken uniformly over all $\mu$ because there are finitely many such $\mu$ for a given economy.

$u \in [0,1]$ so that $\hat{z}_s(u) = u\Delta_s + f(\delta_s^\omega)$ for each school $s$, and $\hat{z}(u) = \big(\hat{z}_s(u)\big)_{s\in S}$,

$$\begin{aligned}\mathcal{I}\big(f(\delta^\mu)\big) - \mathcal{I}\big(f(\delta^\omega)\big) &= \int_{\mathcal{I}\left(f(\delta^\omega)\right)}^{\mathcal{I}\left(f(\delta^\mu)\right)} d\mathcal{I} = \sum_{s\in S}\int_0^1 \Delta_s \frac{\partial \mathcal{I}}{\partial z_s}\big(\hat{z}(u)\big)du \\ &= \sum_{s\in L^{\mathcal{I},f}} \Delta_s \int_0^1 \frac{\partial \mathcal{I}}{\partial z_s}\big(\hat{z}(u)\big)du - \sum_{s\in H^{\mathcal{I},f}} |\Delta_s| \int_0^1 \frac{\partial \mathcal{I}}{\partial z_s}\big(\hat{z}(u)\big)du \\ &\leq 0,\end{aligned}$$

where the second line equality follows from the fact that $\Delta_s \leq 0$ for each school $s \in H^{\mathcal{I},f}$; and the last line inequality is true because the set $L^{\mathcal{I},f}$ includes all schools with a negative partial derivative of $\mathcal{I}$ with respect to their statistic value on the relevant domain, $H^{\mathcal{I},f}$ includes all schools with a positive partial derivative of $\mathcal{I}$ with respect to their statistic value on the relevant domain, and $\Delta_s \geq 0$ for each school $s \in L^{\mathcal{I},f}$.

## A.3 Proposition 1 for Inequality Indices which Depend on Market Sizes

Inequality indices may naturally depend on the base economy and the replica economies for each size $n$ through the weights $(w_s^n)_s$ assigned to schools. This is the case, for instance, for the weighted Gini index defined as

$$\mathcal{G}(z) := \frac{1}{2\sum_{s\in S} w_s z_s}\sum_{s\in S}\sum_{s'\in S} |z_s - z_{s'}|\, w_s^n\, w_{s'}^n$$

which depends on the weights $w_s^n = \frac{|E_s^n|+q_s}{\sum_{s'\in S}|E_{s'}^n|+q_{s'}}$ associated to each school $s$. Clearly, along a sequence of $n$-economies, those weights vary.

To capture the dependence of an inequality index to the market size through the weights, let us redefine an **inequality index** as a function $\mathcal{I} : \mathbb{R}^{|S|} \times [0,1]^{|S|} \to \mathbb{R}$. Fixing a base economy $\langle T, \Theta, \tau, S, q, P, \omega; E^1, \mathcal{V}\rangle$, in the $n$-economy $E^n$, we denote the value taken by the index when the value of the statistic is $z$ and the **vector of weights** is $w^n = (w_s^n)_{s\in S} \in [0,1]^{|S|}$ by $\mathcal{I}(z, w^n)$. We sometimes refer to $\{w^n\}_{n\geq 1}$ as the sequence of weights **associated to inequality index** $\mathcal{I}$. We restrict our attention to sequences $\{w^n\}_{n\geq 1}$ which converge and denote the limit point by $w^*$. We maintain our assumption that an inequality index is continuously differentiable in each $z_s$ argument on an open and dense set. Formally, let $\mathcal{O} := \{(z,w) \in \mathbb{R}^{|S|} \times [0,1]^{|S|} :$ for each $s$, $\frac{\partial \mathcal{I}}{\partial z_s}(z,w)$ exists and is continuous in both arguments$\}$. Our assumption is that $\mathcal{O}$ is open and dense in $\mathbb{R}^{|S|} \times [0,1]^{|S|}$. In this context, genericity is defined as follows: For a given inequality index $\mathcal{I}$, associated sequence of weights $\{w^n\}_{n\geq 1}$ and statistic $f$, the base economy is **generic** if $(z^{*f}, w^*) \in \mathcal{O}$ and $\frac{\partial \mathcal{I}}{\partial z_s}(z^{*f}, w^*) \neq 0$ for all $s$.

Given inequality index $\mathcal{I}$, associated sequence of weights $\{w^n\}_{n\geq 1}$ and statistic $f$, we define $L^{\mathcal{I},w^*,f}$ as the set of schools $s$ with $\frac{\partial \mathcal{I}}{\partial z_s}(z^{*f}, w^*) \leq 0$ and, similarly, $H^{\mathcal{I},w^*,f}$ stands for the set of schools $s$ with $\frac{\partial \mathcal{I}}{\partial z_s}(z^{*f}, w^*) > 0$. The **natural type ranking profile** is defined as the type ranking profile induced by partition $(L^{\mathcal{I},w^*,f}, H^{\mathcal{I},w^*,f})$.

While this involves no substantial changes, for the sake of completeness, we now state and prove Proposition 1 in a context where the inequality index may depend on $n$ (the size of the economy) through its weights.

**Proposition A.1.** *Consider an inequality index $\mathcal{I}$, a continuous and LD-increasing statistic $f$, a generic base economy with status-quo matching of the participants $\omega$, and an associated sequence of weights $\{w^n\}_{n\geq 1}$. If in an $n$-economy with large enough $n$, a matching $\mu$ is status-quo improving for schools under the natural type ranking profile of $\mathcal{I}$, associated weights $\{w^n\}_{n\geq 1}$, and $f$, then*

$$\mathcal{I}\big(f(\delta^{\mu,n}), w^n\big) \leq \mathcal{I}\big(f(\delta^{\omega,n}), w^n\big).$$

**Proof.** Fix an inequality index $\mathcal{I}$, a continuous and LD-increasing statistic $f$, a generic base economy $\langle T, \Theta, \tau, S, q, P, \omega; E^1, \mathcal{V}\rangle$, and an associated sequence of weights $\{w^n\}_{n\geq 1}$. Let us start by defining, for each $n$-economy $E^n$, the *relevant domain of $\mathcal{I}$* for the statistic $f$ as

$$\mathcal{Z}^{f,n} = co\left\{f(\delta^{\mu,n}) : \mu \in \mathcal{M}\right\}.$$

Since genericity holds, by definition, we have that $\frac{\partial \mathcal{I}}{\partial z_s}(z^{*f}, w^*) \neq 0$ for each school $s$ and, in addition, $(z^{*f}, w^*) \in \mathcal{O}$. Since $\mathcal{O}$ is an open set, there is $\varepsilon' > 0$ small enough such that $B_{\varepsilon'}(z^{*f}) \times B_{\varepsilon'}(w^*) \subset \mathcal{O}$. Hence, for each $(z, w) \in B_{\varepsilon'}(z^{*f}) \times B_{\varepsilon'}(w^*)$, $\frac{\partial \mathcal{I}}{\partial z_s}(z, w)$ exists and is continuous in both arguments. Since partial derivative of $\mathcal{I}$ with respect to $z_s$ for each school $s$ at $(z^{*f}, w^*)$ is non-zero, by continuity of $\frac{\partial \mathcal{I}}{\partial z_s}(\cdot,\cdot)$ in both arguments at $(z^{*f}, w^*)$, there must be $\varepsilon \in (0, \varepsilon')$ such that for each school $s$, either

$$\frac{\partial \mathcal{I}}{\partial z_s}(z, w) > 0 \text{ for each } (z, w) \in B_\varepsilon(z^{*f}) \times B_\varepsilon(w^*)$$

or

$$\frac{\partial \mathcal{I}}{\partial z_s}(z, w) < 0 \text{ for each } (z, w) \in B_\varepsilon(z^{*f}) \times B_\varepsilon(w^*).$$

The set of schools for each of which the former inequality holds is $H^{\mathcal{I},w^*,f}$ while the set of schools for each of which the latter one holds is $L^{\mathcal{I},w^*,f}$. Note that, in our sequence of replica economies, for each school $s$, $\delta_s^{\mu,n}$ converges to $\delta_s^*$. Therefore, by continuity of statistic $f$, for $n$ large enough, $f(\delta^{\mu,n}) = \big(f(\delta_s^{\mu,n})\big)_{s\in S} \in B_\varepsilon(z^{*f})$ for each matching $\mu$.[71] Since $B_\varepsilon(z^{*f})$ is convex, this also implies that convex combination of the points $\{f(\delta^{\mu,n}) : \mu \in \mathcal{M}\}$ is in $B_\varepsilon(z^{*f})$. Hence, for any profile of weights $w$, the relevant domain $\mathcal{Z}^{f,n}$ of $\mathcal{I}(\cdot, w)$ in an $n$-economy is included in $B_\varepsilon(z^{*f})$.[72] Further, given that $w^n$ converges

[71] $\varepsilon > 0$ can be taken uniformly over all $\mu$ because there are finitely many such $\mu$ for a given economy.

[72] Note that the relevant domain of $\mathcal{I}(\cdot, w)$ does not depend on the vector of weights $w$.

to $w^*$, for a large enough $n$, we must have that $w^n \in B_\varepsilon(w^*)$. In the sequel, we fix such a large enough $n$ ensuring that both the relevant domain of $\mathcal{I}(\cdot, w^n)$ in an $n$-economy is included in $B_\varepsilon(z^{*f})$ and $w^n \in B_\varepsilon(w^*)$.

First, this implies that $\mathcal{I}(\cdot, w^n)$ has well-defined partial derivatives everywhere on its relevant domain. Since for each $z$ in this relevant domain, there is an open neighborhood of $z$ in which partial derivatives are well-defined and continuous, this also implies that total derivatives of $\mathcal{I}(\cdot, w^n)$ are well-defined on this relevant domain.

Second, this also implies that, for each school $s \in H^{\mathcal{I},w^*,f}$, $\frac{\partial \mathcal{I}}{\partial z_s}(\cdot, w^n)$ is strictly positive on its relevant domain $\mathcal{Z}^{f,n}$ and for each school $s \in L^{\mathcal{I},w^*,f}$, $\frac{\partial \mathcal{I}}{\partial z_s}(\cdot, w^n)$ is strictly negative on this relevant domain $\mathcal{Z}^{f,n}$.

With these in mind, let $\mu$ be a status-quo improving matching for schools under the natural type ranking profile of $\mathcal{I}$, its associated weights $\{w^n\}_{n\geq 1}$, and $f$. We define $\Delta_s = f(\delta_s^\mu) - f(\delta_s^\omega)$ as the change in the value from the status quo to $\mu$ for each school $s$. Because $\mu$ status-quo improves $\omega$ for natural type ranking profile and $f$ is LD-increasing, we have that $\Delta_s \geq 0$ for each $s \in L^{\mathcal{I},f}$ and $\Delta_s \leq 0$ for each $s \in H^{\mathcal{I},f}$ by Lemma 1. Since $\mathcal{I}(\cdot, w^n)$ is differentiable in subdomain $\mathcal{Z}^{f,n}$, its total differential satisfies $d\mathcal{I}(\cdot, w^n) = \sum_{s\in S} \frac{\partial \mathcal{I}}{\partial z_s}(z, w^n) dz_s$. Then to calculate the total variation in the inequality index from $\omega$ to $\mu$, we obtain the following path integral on the linear path in variable $u \in [0,1]$ so that $\hat{z}_s(u) = u\Delta_s + f(\delta_s^\omega)$ for each school $s$, and $\hat{z}(u) = \big(\hat{z}_s(u)\big)_{s\in S}$,

$$
\begin{aligned}
\mathcal{I}\big(f(\delta^\mu), w^n\big) - \mathcal{I}\big(f(\delta^\omega), w^n\big) &= \int_{\mathcal{I}\left(f(\delta^\omega), w^n\right)}^{\mathcal{I}\left(f(\delta^\mu), w^n\right)} d\mathcal{I}(\cdot, w^n) = \sum_{s\in S} \int_0^1 \Delta_s \frac{\partial \mathcal{I}}{\partial z_s}\big(\hat{z}(u), w^n\big) du \\
&= \sum_{s\in L^{\mathcal{I},f}} \Delta_s \int_0^1 \frac{\partial \mathcal{I}}{\partial z_s}\big(\hat{z}(u), w^n\big) du - \sum_{s\in H^{\mathcal{I},f}} |\Delta_s| \int_0^1 \frac{\partial \mathcal{I}}{\partial z_s}\big(\hat{z}(u), w^n\big) du \\
&\leq 0,
\end{aligned}
$$

where the second line equality follows from the fact that $\Delta_s \leq 0$ for each school $s \in H^{\mathcal{I},w^*,f}$, and the last line inequality is true because the set $L^{\mathcal{I},w^*,f}$ includes all schools with a negative partial derivative of $\mathcal{I}(\cdot, w^n)$ with respect to their statistic value on the relevant domain and $H^{\mathcal{I},w^*,f}$ includes all schools with a positive partial derivative of $\mathcal{I}(\cdot, w^n)$ with respect to their statistic value on the relevant domain, and $\Delta_s \geq 0$ for each school $s \in L^{\mathcal{I},w^*,f}$.

## A.4 Relationship between Regularity and Standard Axioms on Inequality Indices

We show that the regularity of an inequality index is ensured when the index satisfies the three properties defined below. First, say that the index $\mathcal{I}$ satisfies the **strict Pigou-Dalton transfer principle** if the following holds. Fix any $p > 0$ and any two schools $s, s'$

such that $z_s - p > z_{s'} + p$ and let $z'$ be the vector $z$ where $z_s$ is replaced by $z_s - p$ and $z_{s'}$ is replaced by $z_{s'} + p$ (we refer to $p$ as a **Pigou-Dalton transfer**), we must have that $\mathcal{I}(z') < \mathcal{I}(z)$. Further, index $\mathcal{I}$ satisfies **scale invariance**, if for any $\lambda > 0, \mathcal{I}(\lambda z) = \mathcal{I}(z)$. Finally, we say that $\mathcal{I}$ is **responsive** if whenever $\mathcal{I}$ is differentiable at $z$, we have $\frac{\partial \mathcal{I}}{\partial z_s}(z) \neq 0$ for all $s$.

**Proposition A.2.** *Consider a responsive inequality index $\mathcal{I}$ which satisfies the strict Pigou-Dalton transfer principle and scale invariance. Then, index $\mathcal{I}$ is regular.*

**Proof.** Fix a responsive inequality index $\mathcal{I}$ which satisfies the strict Pigou-Dalton transfer principle and scale invariance. Consider $z = (z_{s_1}, ..., z_{s_m})$ where $z_{s_1} \leq ... \leq z_{s_m}$ (and $m := |S|$) and where $\mathcal{I}$ is differentiable. Let $\varepsilon > 0$ and $z^\varepsilon = (z_{s_1}, ..., z_{s_m} + \varepsilon)$. We want to show that $\mathcal{I}(z^\varepsilon) > \mathcal{I}(z)$ which implies that the right derivative of $\mathcal{I}$ at $z$ with respect to $z_{s_m}$ is positive and since $\mathcal{I}$ is differentiable at $z$, this also implies that $\frac{\partial \mathcal{I}(z)}{\partial z_{s_m}} \geq 0$.[73] Clearly, by responsiveness of $\mathcal{I}$, we obtain that $\frac{\partial \mathcal{I}(z)}{\partial z_{s_m}} > 0$. (Note that a symmetric reasoning applies to show that $\frac{\partial \mathcal{I}(z)}{\partial z_{s_1}} < 0$.)

To see that $\mathcal{I}(z^\varepsilon) > \mathcal{I}(z)$, let $\lambda = \frac{\sum_{i=1}^m z_{s_i}}{\sum_{i=1}^m z_{s_i} + \varepsilon}$ and observe that (1) by scale invariance $\mathcal{I}(z^\varepsilon) = \mathcal{I}(\lambda z^\varepsilon)$; (2) we can move from $\lambda z^\varepsilon$ to $z$ by subtracting $\lambda(z_{s_m} + \varepsilon) - z_{s_m} > 0$ from the $m^{\text{th}}$ element of $\lambda z^\varepsilon$ and adding $(1-\lambda) z_{s_i}$ to each $i^{\text{th}}$ element of $\lambda z^\varepsilon$ for $i \neq m$. Note that

$$\begin{aligned}
(\lambda(z_{s_m} + \varepsilon) - z_{s_m}) - (1-\lambda)\left(z_{s_1} + ... + z_{s_{m-1}}\right) &= \lambda\varepsilon - (1-\lambda) z_{s_m} - (1-\lambda)\left(z_{s_1} + ... + z_{s_{m-1}}\right) \\
&= \lambda\varepsilon - (1-\lambda)\left(z_{s_1} + ... + z_{s_{m-1}} + z_{s_m}\right) \\
&= \frac{\sum_{i=1}^m z_{s_i}}{\sum_{i=1}^m z_{s_i} + \varepsilon}\varepsilon - \frac{\varepsilon}{\sum_{i=1}^m z_{s_i} + \varepsilon}\sum_{i=1}^m z_{s_i} \\
&= \varepsilon\left(\frac{\sum_{i=1}^m z_{s_i}}{\sum_{i=1}^m z_{s_i} + \varepsilon} - \frac{\sum_{i=1}^m z_{s_i}}{\sum_{i=1}^m z_{s_i} + \varepsilon}\right) = 0.
\end{aligned}$$

Hence, we can move from $\lambda z^\varepsilon$ to $z$ by a sequence of $m-1$ Pigou-Dalton transfers (i.e., for $i = 1, ...m-1$, transfer $(1-\lambda) z_{s_i}$ from the $m^{\text{th}}$ element of $\lambda z^\varepsilon$ to the $i^{\text{th}}$ element of $\lambda z$). This is a well-defined Pigou-Dalton transfer for $\varepsilon > 0$ small enough given that $z_{s_m} + \varepsilon > z_{s_i}$ for all $i \neq m$). Then, because $\mathcal{I}$ satisfies the strict Pigou-Dalton transfer principle, we obtain that $\mathcal{I}(\lambda z^\varepsilon) > \mathcal{I}(z)$. We conclude that $\mathcal{I}(z^\varepsilon) > \mathcal{I}(z)$, as claimed.

## A.5 Proof of Proposition 2

The following lemma implies Proposition 2.

**Lemma A.1.** *Consider a regular inequality index $\mathcal{I}$ and a continuous and strictly LD-increasing*

[73] This cannot be strengthened to a strict inequality. Indeed, it is well-known that a strictly increasing function at a point can have a derivative equal to 0 at that point.

*statistic $f$. Fix any generic base economy with two schools, no new teachers, no vacant seats, $\Theta = \{\theta_1, \theta_2\}$, and status-quo matching $\omega$. Then, in an $n$-economy with large enough $n$, an individually rational matching $\mu$ status-quo improves schools under the natural type ranking profile of $\mathcal{I}$ and $f$, if and only if $\mathcal{I}\big(f(\delta^{\mu,n})\big) \leq \mathcal{I}\big(f(\delta^{\omega,n})\big)$.*

**Proof.** Given Proposition 1, we only prove the "if part". Fix a regular inequality index $\mathcal{I}$ and a continuous and strictly LD-increasing statistic $f$. Let $S = \{s_-, s_+\}$ be such that $s_-$ is the school $s$ with $\frac{\partial \mathcal{I}}{\partial z_s}(z^{*f}) < 0$ and $s_+$ is the school $s$ with $\frac{\partial \mathcal{I}}{\partial z_s}(z^{*f}) > 0$. These schools are well-defined by genericity and regularity. Therefore, the natural type ranking profile of $\mathcal{I}$ and $f$, $\trianglerighteq = (\trianglerighteq_{s_-}, \trianglerighteq_{s_+})$, over the type set $\Theta = \{\theta_1, \theta_2\}$ satisfies (ignoring $\theta_\emptyset$ without loss of generality as there are no vacant seats)

$$\theta_2 \ \triangleright_{s_-} \ \theta_1 \qquad \text{and} \qquad \theta_1 \ \triangleright_{s_+} \ \theta_2$$

as $L^{\mathcal{I},f} = \{s_-\}$ and $H^{\mathcal{I},f} = \{s_+\}$.

Recall that for each $n$-economy, the *relevant domain of* $\mathcal{I}$ for the statistic $f$ is

$$\mathcal{Z}^{f,n} = co\left\{f(\delta^{\mu,n}) : \mu \in \mathcal{M}\right\}.$$

In the sequel, we let $\mathcal{O}$ be the set of points at which $\mathcal{I}$ is continuously differentiable (which, by assumption, is an open and dense set of values in $\mathbb{R}^{|S|}$). Since $\mathcal{O}$ is an open set, there is $\varepsilon' > 0$ small enough such that $B_{\varepsilon'}(z^{*f}) \subset \mathcal{O}$. Hence, for each $z \in B_{\varepsilon'}(z^{*f})$, $\mathcal{I}$ is continuously differentiable. Since derivatives of $\mathcal{I}$ at $z^{*f}$ are non-zero, by continuity, there must be $\varepsilon \in (0, \varepsilon')$ such that

$$\frac{\partial \mathcal{I}}{\partial z_{s_-}}(z) < 0 \text{ for each } z \in B_\varepsilon(z^{*f}),$$

and

$$\frac{\partial \mathcal{I}}{\partial z_{s_+}}(z) > 0 \text{ for each } z \in B_\varepsilon(z^{*f}).$$

Since our sequence of economies consists of replicas, it is easy to show that, $\delta^{\mu',n}$ converge to a profile of value distributions given by $\delta^* := \left(\frac{|E_s^{\theta_1}|}{|E_s|}, \frac{|E_s^{\theta_2}|}{|E_s|}\right)_{s\in S}$ for any matching $\mu'$. Because $f$ is continuous, for $n$ large enough, $f(\delta^{\mu',n}) = \big(f(\delta_s^{\mu',n})\big)_{s\in S} \in B_\varepsilon(z^{*f})$ for each matching $\mu'$.[74] Since $B_\varepsilon(z^{*f})$ is convex, this also implies that any convex combination of the points $\{f(\delta^{\mu,n}) : \mu \in \mathcal{M}\}$ is in $B_\varepsilon(z^{*f})$. Hence, the relevant domain $\mathcal{Z}^{f,n}$ of $\mathcal{I}$ in an $n$-economy is included in $B_\varepsilon(z^{*,f})$. In the sequel, we fix such a large enough $n$.

Assume that an individually rational matching $\mu$ does not status-quo improve schools with respect to the natural type ranking profile $\trianglerighteq$. We want to show that $\mathcal{I}\big(f(\delta^{\mu,n})\big) >$

[74] $\varepsilon > 0$ can be taken uniformly over all $\mu'$ because there are finitely many such $\mu'$.

$\mathcal{I}\big(f(\delta^{\omega,n})\big)$. That $\mu$ does not status-quo improve schools with respect to the natural type ranking profile $\trianglerighteq$ means that for some school $s \in \{s_-, s_+\}$

1. either $|\mu_s| < |\omega_s|$, or
2. for some $\theta \in \{\theta_1, \theta_2\}$ :

$$\sum_{\theta' \trianglerighteq_s \theta} |\mu_s^{\theta'}| < \sum_{\theta' \trianglerighteq_s \theta} |\omega_s^{\theta'}|.$$

Assume without loss that $s = s_-$. Note first that if $|\mu_{s_-}| < |\omega_{s_-}|$ then for the school $s_+$, we must have $q_{s_+} \geq |\mu_{s_+}| > |\omega_{s_+}|$ since $\mu$ is individually rational.[75] This implies that $s_+$ must have some empty seat initially, a contradiction. Note that this argument shows more generally that $|\mu_s| = |\omega_s|$ for each $s$. Hence, Condition (2) above must hold. Note that with only two types, this implies that $\theta = \theta_2$, i.e.,

$$|\mu_{s_-}^{\theta_2}| < |\omega_{s_-}^{\theta_2}|$$

and so since $|\mu_{s_-}| = |\omega_{s_-}|$, this also implies that

$$|\mu_{s_-}^{\theta_1}| > |\omega_{s_-}^{\theta_1}|.$$

Since for each school $s \in \{s_-, s_+\}$ and each $k = 1, 2$ :

$$\delta_s^{\mu,v_k} = \frac{|E_s^{\theta_k}| + |\mu_s^{\theta_k}|}{|E_s| + q_s},$$

we must also have that

$$\delta_{s_-}^{\mu,v_2} < \delta_{s_-}^{\omega,v_2}$$

and

$$\delta_{s_-}^{\mu,v_1} > \delta_{s_-}^{\omega,v_1}.$$

Similarly, we must have

$$\delta_{s_+}^{\mu,v_2} > \delta_{s_+}^{\omega,v_2}$$

and

$$\delta_{s_+}^{\mu,v_1} < \delta_{s_+}^{\omega,v_1}.$$

With this in our hands, we can define $\Delta_s = f(\delta_s^{\mu}) - f(\delta_s^{\omega})$ as the change in the statistic value from the status quo to $\mu$ for each school $s \in \{s_-, s_+\}$. Because $f$ is strictly LD-increasing, we have that $\Delta_{s_-} < 0$ and $\Delta_{s_+} > 0$. Since, by our assumption that $n$ is large enough, $\mathcal{I}$ is differentiable in subdomain $\mathcal{Z}^f$, its total differential satisfies

[75]Note that individual rationality is needed here. Indeed, consider the following example. There is a teacher $t_1$ with type $\theta_1$ initially assigned school $s_-$. There are teachers $t_1'$ with type $\theta_1$ and $t_2'$ with type $\theta_2$ initially assigned school $s_+$. Let $\mu$ be the matching which simply makes $t_2'$ unassigned. This violates condition 1. in the definition of status-quo improvement and so $\mu$ does not status-quo improve schools. However, we achieve perfect equality across schools.

$d\mathcal{I} = \sum_{s\in S} \frac{\partial \mathcal{I}}{\partial z_s}(z)dz_s$. Then to calculate the total variation in the inequality index from $\omega$ to $\mu$, we obtain the following path integral on the linear path in variable $u \in [0,1]$ so that $\hat{z}_s(u) = u\Delta_s + f(\delta_s^\omega)$ for each school $s \in \{s_-, s_+\}$, and $\hat{z}(u) = \big(\hat{z}_{s_+}(u), \hat{z}_{s_-}(u)\big)$,

$$\begin{aligned}\mathcal{I}\big(f(\delta^\mu)\big) - \mathcal{I}\big(f(\delta^\omega)\big) &= \int_{\mathcal{I}\left(f(\delta^\omega)\right)}^{\mathcal{I}\left(f(\delta^\mu)\right)} d\mathcal{I} = \sum_{s\in S}\int_0^1 \Delta_s \frac{\partial \mathcal{I}}{\partial z_s}\big(\hat{z}(u)\big)du \\ &= \Delta_{s_-}\int_0^1 \frac{\partial \mathcal{I}}{\partial z_s}\big(\hat{z}(u)\big)du + \Delta_{s_+}\int_0^1 \frac{\partial \mathcal{I}}{\partial z_s}\big(\hat{z}(u)\big)du \\ &> 0,\end{aligned}$$

where the last line inequality follows from $\Delta_{s_-} < 0$ and $\Delta_{s_+} > 0$ together with the fact that $\frac{\partial \mathcal{I}}{\partial z_{s_-}}(z) < 0$ on the relevant domain of $\mathcal{I}$ and $\frac{\partial \mathcal{I}}{\partial z_{s_+}}(z) > 0$ on the relevant domain of $\mathcal{I}$. This last fact comes from our assumption that $n$ is large enough. This implies that the value of the inequality index $\mathcal{I}$ strictly increases, i.e.,

$$\mathcal{I}\big(f(\delta^{\mu,n})\big) > \mathcal{I}\big(f(\delta^{\omega,n})\big).$$

## A.6 Proof of Proposition 3

Consider a market with the set of schools $S = \{s_1, s_2, s_3, s_4\}$, (re)assignment quota vector $q = (1,1,1,1)$ and the set of types $\Theta = \{\theta_0, \ell, h\}$ with type values $0 = v_0 < v_\ell < v_h$. We assume schools do not have vacant seats and there are no new teachers (particularly no teachers of the lowest type $\theta_0$). Since effectively there are only two types, a type value distribution for a school is simply a vector $(x,y) \in [0,1] \times [0,1]$ where $x$ is the fraction of teachers of type $\ell$ assigned to the school and $y$ the fraction of teachers of type $h$ (we omit vacant seats since there are none). In what follows, $f(x,y)$ will be the value of the statistic for the type value distribution $(x,y)$. Suppose the status-quo matching $\omega$ and set of profiles of teachers who do not participate in the (re)assignment $E^n$ for some $n$ are as follows (where the letter name of the teacher also denotes the type of the teacher):

$$\begin{aligned}\omega_{s_1} &= \{\ell_1\} &\quad\text{and}\quad& E^n_{s_1} = \{\ell_1^1, \ldots, \ell_1^n\},\\ \omega_{s_2} &= \{\ell_2\} &\quad\text{and}\quad& E^n_{s_2} = \{\ell_2^1, \ldots, \ell_2^n\},\\ \omega_{s_3} &= \{h_3\} &\quad\text{and}\quad& E^n_{s_3} = \{h_3^1, \ldots, h_3^n\},\\ \omega_{s_4} &= \{h_4\} &\quad\text{and}\quad& E^n_{s_4} = \{h_4^1, \ldots, h_4^n\}.\end{aligned}$$

While our variables depend on $n$, we do not explicitly note the dependence to $n$ whenever this is clear from the context.

Recall that, since $\mathcal{I}$ is regular and $\{s_1, s_2\} = \arg\min_s z_s^{*f}$ and $\{s_3, s_4\} = \arg\max_s z_s^{*f}$, we must have $\frac{\partial \mathcal{I}}{\partial z_s}(z^{*f}) < 0$ for each $s \in \{s_1, s_2\}$ while $\frac{\partial \mathcal{I}}{\partial z_s}(z^{*f}) > 0$ for each $s \in \{s_3, s_4\}$ where—as each school in $\{s_1, s_2\}$ has only type $\ell$ non-participating employees and each school in $\{s_3, s_4\}$ has only type $h$ non-participating employees —

$$z^{*f} = \big(f(1,0), f(1,0), f(0,1), f(0,1)\big).$$

Thus,

$$L^{\mathcal{I},f} := \left\{ s \in S \;:\; \frac{\partial \mathcal{I}}{\partial z_s}(z^{*f}) \leq 0 \right\} = \{s_1, s_2\},$$

while

$$H^{\mathcal{I},f} := \left\{ s \in S \;:\; \frac{\partial \mathcal{I}}{\partial z_s}(z^{*f}) > 0 \right\} = \{s_3, s_4\}.$$

Partition $\big(L^{\mathcal{I},f}, H^{\mathcal{I},f}\big)$ induces the natural type ranking profile $\trianglerighteq$ where

1. for each school $s \in L^{\mathcal{I},f}$, $\;h \;\triangleright_s\; \ell \;\triangleright_s\; \theta_0 \;\triangleright_s\; \theta_\emptyset$, and
2. for each school $s \in H^{\mathcal{I},f}$, $\;\theta_0 \;\triangleright_s\; \theta_\emptyset \;\triangleright_s\; \ell \;\triangleright_s\; h$.

Suppose the preference profile $P$ of the participating teachers is given as

$$\begin{aligned}
\ell_1 &: s_3 \; P_{\ell_1} \; s_1,\\
\ell_2 &: s_4 \; P_{\ell_2} \; s_2,\\
h_3 &: s_2 \; P_{h_3} \; s_1 \; P_{h_3} \; s_3,\\
h_4 &: s_2 \; P_{h_4} \; s_1 \; P_{h_4} \; s_4.
\end{aligned}$$

There are four individually rational matchings besides $\omega$:

$$\begin{array}{llll}
\mu^1_{s_1} = \{h_3\}, & \mu^1_{s_2} = \{h_4\}, & \mu^1_{s_3} = \{\ell_1\}, & \mu^1_{s_4} = \{\ell_2\}.\\
\mu^2_{s_1} = \{h_4\}, & \mu^2_{s_2} = \{h_3\}, & \mu^2_{s_3} = \{\ell_1\}, & \mu^2_{s_4} = \{\ell_2\}.\\
\mu^3_{s_1} = \{h_3\}, & \mu^3_{s_2} = \{\ell_2\}, & \mu^3_{s_3} = \{\ell_1\}, & \mu^1_{s_4} = \{h_4\}.\\
\mu^4_{s_1} = \{\ell_1\}, & \mu^4_{s_2} = \{h_4\}, & \mu^4_{s_3} = \{h_3\}, & \mu^4_{s_4} = \{\ell_2\}.
\end{array}$$

We consider the SI-CC mechanism induced by the natural type ranking profile $\trianglerighteq$. One can check that SI-CC yields matching $\mu^1$. It should be clear that $\mu^1$ and $\mu^2$ are the matchings which yield the smallest value for the inequality index $\mathcal{I}$. More generally, let us prove that

$$\mathcal{I}\big(f(\delta^{\mu^1})\big) = \mathcal{I}\big(f(\delta^{\mu^2})\big) < \mathcal{I}\big(f(\delta^{\mu^3})\big) = \mathcal{I}\big(f(\delta^{\mu^4})\big) < \mathcal{I}\big(f(\delta^{\omega})\big). \tag{18}$$

The first equality is straightforward while the second one holds by symmetry of $\mathcal{I}$. As for the inequalities, let us start with the first strict inequality, i.e.,

$$\mathcal{I}\big(f(\delta^{\mu^1})\big) < \mathcal{I}\big(f(\delta^{\mu^3})\big).$$

Consider changing the matching from $\mu^3$ to $\mu^1$. We denote the change in the value of the statistic at a school $s$ by $\Delta_s$. First, the value of the statistic does not vary for $s_1$ and $s_3$. For school $s_2$, it varies by $\Delta_{s_2} = f(1-\frac{1}{n+1}, \frac{1}{n+1}) - f(1,0) > 0$ and for school $s_4$ it varies by $\Delta_{s_4} = f(\frac{1}{n+1}, 1-\frac{1}{n+1}) - f(0,1) < 0$ (the strict inequalities use our assumption that $f$ is strictly LD-increasing).

Since $\mathcal{I}$ is differentiable everywhere, its total differential satisfies $d\mathcal{I} = \sum_{s\in S} \frac{\partial \mathcal{I}}{\partial z_s}(z)dz_s$. Then to calculate the total variation in the inequality index from $\mu^3$ to $\mu^1$, we obtain the following path integral on the linear path in variable $u \in [0,1]$ so that $\hat{z}_s(u) = u\Delta_s + f(\delta_s^{\mu^3})$ for each school $s$, and $\hat{z}(u) = \big(\hat{z}_s(u)\big)_{s\in S}$,

$$\begin{aligned}\mathcal{I}\big(f(\delta^{\mu^1})\big) - \mathcal{I}\big(f(\delta^{\mu^3})\big) &= \int_{\mathcal{I}\left(f(\delta^{\mu^3})\right)}^{\mathcal{I}\left(f(\delta^{\mu^1})\right)} d\mathcal{I} = \int_0^1 \sum_{s\in S} \Delta_s \frac{\partial \mathcal{I}}{\partial z_s}\big(\hat{z}(u)\big)du \\ &= \int_0^1 \left[\Delta_{s_2}\frac{\partial \mathcal{I}}{\partial z_{s_2}}\big(\hat{z}(u)\big) + \Delta_{s_4}\frac{\partial \mathcal{I}}{\partial z_{s_4}}\big(\hat{z}(u)\big)\right] du \\ &< 0,\end{aligned}$$

where the strict inequality uses the facts that $\Delta_{s_2} > 0$ and $\Delta_{s_4} < 0$, and also $\frac{\partial \mathcal{I}}{\partial z_{s_2}}\big(\hat{z}(u)\big) < 0$ and $\frac{\partial \mathcal{I}}{\partial z_{s_4}}\big(\hat{z}(u)\big) > 0$ for each $u \in [0,1)$. To see why the latter is true, observe that, by definition,

$$\begin{aligned}\hat{z}_{s_2}(u) &= uf\left(1-\frac{1}{n+1}, \frac{1}{n+1}\right) + (1-u)f(1,0), \\ \hat{z}_{s_1}(u) &= f\left(1-\frac{1}{n+1}, \frac{1}{n+1}\right) \\ \hat{z}_{s_3}(u) &= f\left(\frac{1}{n+1}, 1-\frac{1}{n+1}\right)\end{aligned}$$

while

$$\hat{z}_{s_4}(u) = uf\left(\frac{1}{n+1}, 1-\frac{1}{n+1}\right) + (1-u)f(0,1).$$

Since $f$ is strictly LD-increasing, $f(0,1) > f\left(\frac{1}{n+1}, 1-\frac{1}{n+1}\right) > f\left(1-\frac{1}{n+1}, \frac{1}{n+1}\right) > f(1,0)$, we must have that $\hat{z}_{s_2}(u) < \hat{z}_s(u) < \hat{z}_{s_4}(u)$ for each $s \in \{s_1, s_3\}$ and $u \in [0,1)$. By assumption, we must have $\frac{\partial \mathcal{I}}{\partial z_{s_2}}\big(\hat{z}(u)\big) < 0$ for each $u \in [0,1)$ while $\frac{\partial \mathcal{I}}{\partial z_{s_4}}\big(\hat{z}(u)\big) > 0$ for each $u \in [0,1)$. Hence, we must have

$$\mathcal{I}\big(f(\delta^{\mu^1})\big) < \mathcal{I}\big(f(\delta^{\mu^3})\big).$$

Now, to complete the proof for Equation (18), let us show the inequality

$$\mathcal{I}\big(f(\delta^{\mu^4})\big) < \mathcal{I}\big(f(\delta^{\omega})\big).$$

Consider changing the matching from $\omega$ to $\mu^4$. Again, let us denote the change in the value of the statistic at school $s$ by $\Delta_s$. First, the value of the statistic does not vary for $s_1$

and $s_3$. For school $s_2$, it varies by $\Delta_{s_2} = f(1 - \frac{1}{n+1}, \frac{1}{n+1}) - f(1,0) > 0$ and for school $s_4$ it varies by $\Delta_{s_4} = f(\frac{1}{n+1}, 1 - \frac{1}{n+1}) - f(0,1) < 0$ (the strict inequalities use our assumption that $f$ is strictly LD-increasing). By a similar argument as above, we have that the total variation in the inequality index from $\omega$ to $\mu^4$ corresponds to the following path integral on the linear path in variable $u \in [0,1]$ so that $\bar{z}_s(u) = u\Delta_s + f(\delta_s^\omega)$ for each school $s$, and $\bar{z}(u) = \big(\bar{z}_s(u)\big)_{s\in S}$,

$$\begin{aligned}\mathcal{I}\big(f(\delta^{\mu^4})\big) - \mathcal{I}\big(f(\delta^\omega)\big) &= \int_{\mathcal{I}\left(f(\delta^\omega)\right)}^{\mathcal{I}\left(f(\delta^{\mu^4})\right)} d\mathcal{I} = \int_0^1 \sum_{s\in S} \Delta_s \frac{\partial \mathcal{I}}{\partial z_s}\big(\bar{z}(u)\big) du \\ &= \int_0^1 \left[\Delta_{s_2} \frac{\partial \mathcal{I}}{\partial z_{s_2}}\big(\bar{z}(u)\big) + \Delta_{s_4} \frac{\partial \mathcal{I}}{\partial z_{s_4}}\big(\bar{z}(u)\big)\right] du \\ &< 0.\end{aligned}$$

The inequality is proved as follows. Since $\delta^{\mu^4,n}, \delta^{\omega,n}$ converge to the profile of value distributions $\big((1,0),(1,0),(0,1),(0,1)\big)$, and because statistic $f$ is continuous, we have

$$f(\delta^{\mu^4,n}) \to z^{*f} \text{ and } f(\delta^{\omega,n}) \to z^{*f}.$$

Given that $\frac{\partial \mathcal{I}}{\partial z_{s_2}}(z^{*f}) < 0$ and $\frac{\partial \mathcal{I}}{\partial z_{s_4}}(z^{*f}) > 0$, by continuous differentiability, there exists $\varepsilon > 0$ such that any $z \in B_\varepsilon(z^{*f})$ satisfies $\frac{\partial \mathcal{I}}{\partial z_{s_2}}(z) < 0$ and $\frac{\partial \mathcal{I}}{\partial z_{s_4}}(z) > 0$. From now on, we assume that $n$ is large enough so that $f(\delta^{\mu^4,n})$ and $f(\delta^{\omega,n})$ are both in $B_\varepsilon(z^{*f})$. Since $\bar{z}(u)$ is a convex combination of $f(\delta^{\mu^4,n})$ and $f(\delta^{\omega,n})$ and $B_\varepsilon(z^{*f})$ is convex, we have that $\bar{z}(u) \in B_\varepsilon(z^{*f})$. Since any point in $B_\varepsilon(z^{*f})$ has the same sign for derivatives as at $z^{*f}$, we conclude that for $n$ large enough,

$$\frac{\partial \mathcal{I}}{\partial z_{s_2}}\big(\bar{z}(u)\big) < 0 \text{ and } \frac{\partial \mathcal{I}}{\partial z_{s_4}}\big(\bar{z}(u)\big) > 0,$$

for each $u \in [0,1]$. Hence, in such a case

$$\begin{aligned}\mathcal{I}\big(f(\delta^{\mu^4})\big) - \mathcal{I}\big(f(\delta^\omega)\big) &= \int_0^1 \left[\Delta_{s_2} \frac{\partial \mathcal{I}}{\partial z_{s_2}}\big(\bar{z}(u)\big) + \Delta_{s_4} \frac{\partial \mathcal{I}}{\partial z_{s_4}}\big(\bar{z}(u)\big)\right] du \\ &\leq \int_0^1 \left[\Delta_{s_2} \frac{\partial \mathcal{I}}{\partial z_{s_4}}\big(\bar{z}(u)\big) + \Delta_{s_4} \frac{\partial \mathcal{I}}{\partial z_{s_4}}\big(\bar{z}(u)\big)\right] du \\ &< 0.\end{aligned}$$

We conclude that Equation (18) holds.

Now, let us consider a mechanism $\varphi$ which is individually rational, strategy-proof and has less inequality than SI-CC whenever possible. Because at preference profile $P$, SI-CC selects $\mu^1$, and $\varphi$ has less inequality than SI-CC, by Equation (18), $\varphi$ must select either $\mu^1$ or $\mu^2$ at $P$. First, suppose $\varphi(P) = \mu^1$. When $h_3$ submits an alternative preference relation deeming $s_1$ unacceptable,

$$h_3 : s_2 \; P'_{h_3} \; s_3,$$

she will be better off under $\varphi$. To see this, observe that $h_3$ will no longer be matched with $s_1$ in an individually rational matching for the new preference profile $(P'_{h_3}, P_{-h_3})$. Now, under SI-CC, one can check that $h_3$ remains at $s_3$ and the matching achieved is $\mu^4$. Further, by Equation (18), the unique individually rational matching with strictly less inequality than $\mu^4$ is matching $\mu^2$. Therefore, we should have

$$\varphi\left(P'_{h_3}, P_{-h_3}\right) = \mu^2.$$

Thus, teacher $h_3$ receives her first choice post at $s_2$ by omitting $s_1$ from her list, and she successfully manipulates $\varphi$ if $\varphi(P) = \mu^1$, a contradiction with the strategy-proofness of $\varphi$.

Second, suppose $\varphi(P) = \mu^2$. When $h_4$ submits an alternative preference relation deeming $s_1$ unacceptable,

$$h_4 : s_2 \; P'_{h_4} \; s_4,$$

she will be better off. To see this, observe that $h_4$ will no longer be matched with $s_1$ in an individually rational matching for the new preference profile $(P'_{h_4}, P_{-h_4})$. Now, under SI-CC, one can check that the selected matching remains $\mu^1$. Further, by Equation (18), all other individually rational matchings have strictly more inequality than $\mu^1$. Therefore, we should have

$$\varphi\left(P'_{h_4}, P_{-h_4}\right) = \mu^1.$$

Thus, teacher $h_4$ receives her first choice post at $s_2$ by omitting $s_1$ from her list, and she successfully manipulates $\varphi$ if $\varphi(P) = \mu^2$, a contradiction with the strategy-proofness of $\varphi$.

## A.7 Proposition 1 without Genericity

In the sequel, we assume that $\mathcal{I}$ is continuously differentiable at $z^{*f}$ but we do not assume that the derivatives $\frac{\partial \mathcal{I}}{\partial z_s}(z^{*f})$ are non-zero. Now, we say that $\trianglerighteq$ is the natural type ranking of $\mathcal{I}$ and $f$, if it is induced by partition $(L^{\mathcal{I},f}, H^{\mathcal{I},f})$ where

$$L^{\mathcal{I},f} = \left\{ s \in S \; : \; \frac{\partial \mathcal{I}}{\partial z_s}(z^{*f}) \leq 0 \right\} \quad \text{and} \quad H^{\mathcal{I},f} = \left\{ s \in S \; : \; \frac{\partial \mathcal{I}}{\partial z_s}(z^{*f}) > 0 \right\}.$$

Schools with $\frac{\partial \mathcal{I}}{\partial z_s}(z^{*f})$ equal to 0 at $z^{*f}$ are typically schools for which a small increase in $z_s^{*f}$ may make the derivative $\frac{\partial \mathcal{I}}{\partial z_s}(z^{*f}) > 0$ while a small decrease may make $\frac{\partial \mathcal{I}}{\partial z_s}(z^{*f}) < 0$. However, since, in a large economy, these are schools with derivatives close to 0 on the relevant domain of $\mathcal{I}$, they only have a marginal impact on the value achieved by the inequality index. (Note then that it does not matter whether those are in $L^{\mathcal{I},f}$ or $H^{\mathcal{I},f}$, so we took the convention to include them in $L^{\mathcal{I},f}$.)

If a matching $\mu$ changes the assignment only for schools with a partial derivative of $\mathcal{I}$

with respect to their statistic value equal to zero at $z^{*f}$ then it's clear that the impact on the inequality index will be small in a large economy. Thus, we say that matching $\mu$ is **reactive** if $\delta_s^\mu \neq \delta_s^\omega$ for at least one school $s$ with non-zero derivatives. The term "reactive" simply comes from the fact that a matching which violates this condition has only a marginal impact on the value of the inequality index in a large economy.

Now, we can state a version of Proposition 1 without the genericity requirement.

**Proposition A.3.** *Consider a regular inequality index $\mathcal{I}$ which is continuously differentiable at $z^{*f}$ and a continuous and strictly LD-increasing statistic $f$. Fix a (potentially non-generic) base economy with status-quo matching of the participants $\omega$. If in an $n$-economy with large enough $n$, a reactive matching $\mu$ status-quo improves schools with respect to the natural type ranking profile of $\mathcal{I}$ and $f$, then*

$$\mathcal{I}\big(f(\delta^{\mu,n})\big) \leq \mathcal{I}\big(f(\delta^{\omega,n})\big).$$

**Proof.** Fix a regular inequality index $\mathcal{I}$ which is continuously differentiable at $z^{*f}$ and a continuous and strictly LD-increasing statistic $f$. Also fix a base economy $\langle T, \Theta, \tau, S, q, P, \omega; E, \mathcal{V}\rangle$. Because $\mathcal{I}$ is differentiable at $z^{*f}$, the following sets form a partition of $S$ : $\tilde{L} := \{s : \frac{\partial \mathcal{I}}{\partial z_s}(z^{*f}) < 0\}$, $\tilde{M} := \{s : \frac{\partial \mathcal{I}}{\partial z_s}(z^{*f}) = 0\}$ and $\tilde{H} := \{s : \frac{\partial \mathcal{I}}{\partial z_s}(z^{*f}) > 0\}$. Further, by regularity, $\tilde{L}$ and $\tilde{H}$ are non-empty. Pick any $\beta_* < 0$ satisfying $\beta_* > \max_{s\in\tilde{L}} \frac{\partial \mathcal{I}}{\partial z_s}(z^{*f})$. Similarly, pick any $\beta^* > 0$ satisfying $\beta^* < \min_{s\in\tilde{H}} \frac{\partial \mathcal{I}}{\partial z_s}(z^{*f})$.

Let $\mu$ be a reactive matching that status-quo improves schools under the natural type ranking profile of $\mathcal{I}$ and $f$.

Define $\Delta_s = f(\delta_s^\mu) - f(\delta_s^\omega)$ as the change in the statistic value from the status quo to $\mu$ for each school $s$. Observe that because $\mu$ status-quo improves $\omega$ for the natural type ranking profile of $\mathcal{I}$ and $f$, and $f$ is LD-increasing, we have that $\Delta_s \geq 0$ for each $s \in \tilde{L} \cup \tilde{M} = L^{\mathcal{I},f}$ and $\Delta_s \leq 0$ for each $s \in \tilde{H} = H^{\mathcal{I},f}$ by Lemma 1. Since $\mu$ is reactive and $f$ is strictly LD-increasing, either $\Delta_s > 0$ for some $s \in \tilde{L}$ or $\Delta_s < 0$ for some $s \in \tilde{H}$. Now, fix $\rho > 0$ such that

$$\beta_* \sum_{s\in\tilde{L}} \Delta_s + \sum_{s\in\tilde{M}} \Delta_s \rho - \beta^* \sum_{s\in\tilde{H}} |\Delta_s| < 0. \tag{19}$$

This is well-defined since, from our previous observation, the first and the last term in the expression are non-positive and one of them is strictly negative. Because $\mathcal{I}$ is continuously differentiable, there must be $\varepsilon > 0$ such that for each $z \in B_\varepsilon(z^{*f})$,

$$\frac{\partial \mathcal{I}}{\partial z_s}(z) > \beta^* \text{ for each } s \in \tilde{H},$$

$$\frac{\partial \mathcal{I}}{\partial z_s}(z) \in (-\rho, \rho) \text{ for each } s \in \tilde{M},$$

and

$$\frac{\partial \mathcal{I}}{\partial z_s}(z) < \beta_* \text{ for each } s \in \tilde{L}.$$

Recall that, in our sequence of replica economies, for each school $s$, $\delta_s^{\mu,n}$ converges to $\delta_s^*$. Therefore, by continuity of statistic $f$, for $n$ large enough, $f(\delta^{\mu',n}) = \big(f(\delta_s^{\mu',n})\big)_{s\in S} \in B_\varepsilon(z^{*f})$ for each matching $\mu'$.[76] Since $B_\varepsilon(z^{*f})$ is convex, this also implies that convex combination of the points $\{f(\delta^{\mu,n}) : \mu \in \mathcal{M}\}$ is in $B_\varepsilon(z^{*f})$. Hence, the relevant domain

$$\mathcal{Z}^{f,n} = co\left\{f(\delta^{\mu',n}) : \mu' \in \mathcal{M}\right\}$$

of $\mathcal{I}$ in an $n$-economy is included in $B_\varepsilon(z^{*f})$ for large enough $n$. In the sequel, we fix such a large enough $n$. This implies that, for each school $s \in \tilde{H}$, $\frac{\partial \mathcal{I}}{\partial z_s}$ is greater than $\beta^* > 0$ on its relevant domain $\mathcal{Z}^{f,n}$, for each school $s \in \tilde{M}$, $\frac{\partial \mathcal{I}}{\partial z_s}$ is in $(-\rho, \rho)$ on its relevant domain $\mathcal{Z}^{f,n}$ and for each school $s \in \tilde{L}$, $\frac{\partial \mathcal{I}}{\partial z_s}$ is smaller than $\beta_* < 0$ on this relevant domain $\mathcal{Z}^{f,n}$.

Now, since $\mathcal{I}$ is differentiable, its total differential satisfies $d\mathcal{I} = \sum_{s\in S} \frac{\partial \mathcal{I}}{\partial z_s}(z)dz_s$. Then to calculate the total variation in the inequality index $\mathcal{I}$ from $\omega$ to $\mu$, we obtain the following path integral on the linear path in variable $u \in [0,1]$ so that $\hat{z}_s(u) = u\Delta_s + f(\delta_s^\omega)$ for each school $s$, and $\hat{z}(u) = \big(\hat{z}_s(u)\big)_{s\in S}$,

$$\begin{aligned}
\mathcal{I}\big(f(\delta^\mu)\big) - \mathcal{I}\big(f(\delta^\omega)\big) &= \int_{\mathcal{I}\left(f(\delta^\omega)\right)}^{\mathcal{I}\left(f(\delta^\mu)\right)} d\mathcal{I} = \sum_{s\in S}\int_0^1 \Delta_s \frac{\partial \mathcal{I}}{\partial z_s}\big(\hat{z}(u)\big)du \\
&= \sum_{s\in\tilde{L}} \Delta_s \int_0^1 \frac{\partial \mathcal{I}}{\partial z_s}\big(\hat{z}(u)\big)du + \sum_{s\in\tilde{M}} \Delta_s \int_0^1 \frac{\partial \mathcal{I}}{\partial z_s}\big(\hat{z}(u)\big)du - \sum_{s\in\tilde{H}} |\Delta_s| \int_0^1 \frac{\partial \mathcal{I}}{\partial z_s}\big(\hat{z}(u)\big)du \\
&\leq \beta_* \sum_{s\in\tilde{L}} \Delta_s + \sum_{s\in\tilde{M}} \Delta_s\rho - \beta^* \sum_{s\in\tilde{H}} |\Delta_s| \\
&< 0
\end{aligned}$$

where the second line equality follows from the fact that $\Delta_s \leq 0$ for each school $s \in \tilde{H} = H^{\mathcal{I},f}$; the weak inequality is true because the set $\tilde{L}$ includes all schools with a negative partial derivative of $\mathcal{I}$ with respect to their statistic value smaller than $\beta_*$ on the relevant domain, $\tilde{H}$ includes all schools with a positive partial derivative of $\mathcal{I}$ greater than $\beta^*$ with respect to their statistic value on the relevant domain while $\tilde{M}$ includes all schools with a partial derivative of $\mathcal{I}$ with respect to their statistic value on the relevant domain that are in in $(-\rho, \rho)$; the strict inequality holds true by construction of $\rho$ in Equation (19).

## A.8 Proposition 3 for Gini and Other Indices That are not Differentiable Everywhere

We explain how to obtain the conclusion of Proposition 3 for inequality indices that may not be differentiable everywhere such as the Gini index.

[76] Recall that there are finitely many such $\mu'$ for a given economy.

First, let us make a technical remark. So far a value distribution for a given school $s$ was a point in $\Delta^{K-1}$ and the support was fixed to the set of values $\mathcal{V}$. However, in the sequel, we will sometimes consider sequences of value distributions with varying support. Hence, we see a value distribution $\delta_s$ as a distribution (with mass points) in the larger space $\Delta(\mathbb{R}_+)$ and use weak convergence of measure to define our topology on this space. A profile of value distributions $\delta$ is a member of $\Delta(\mathbb{R}_+)^{|S|}$ and we simply use the product topology on this product space.

With this topology, we say that $f$ is **continuous** if $\delta^n \to \delta^*$ implies $f(\delta^n) \to f(\delta^*)$. We also say that statistic $f$ is **non-trivial** if for any value distribution $\delta_s$ which puts full mass on $v > 0$ (for any school $s$), we have $f(\delta_s) > 0$. We focus on the weighted Gini index defined as

$$\mathcal{G}(z) := \frac{1}{2\sum_{s\in S} w_s z_s} \sum_{s\in S} \sum_{s'\in S} |z_s - z_{s'}|\, w_s\, w_{s'}$$

which is differentiable everywhere but at $\{z : z_s = z_{s'} \text{ for some } s \neq s'\}$. We assume that weight $w_s := \frac{|E_s|+q_s}{\sum_{s'\in S} |E_{s'}|+q_{s'}}$ for each school $s$.

**Proposition A.4.** *Consider the Gini index $\mathcal{G}$. Fix any strictly LD-increasing, continuous and non-trivial statistic $f$. There is no individually rational and strategy-proof mechanism that has less inequality when possible than SI-CC induced by the natural type ranking profile of $\mathcal{G}$ and $f$.*

**Proof.** Consider a market with the set of schools $S = \{s_1, s_2, s_3, s_4\}$, (re)assignment quota vector $q = (1,1,1,1)$ and the set of types $\Theta = \{\ell, h\}$ with type values $v_0 = 0 < v_\ell < v_h$. Since there are only two types, a type value distribution for a school is simply a vector $(x, y) \in [0,1] \times [0,1]$ where $x$ is the fraction of teachers of type $\ell$ and $y$ the fraction of teachers of type $h$ (we omit vacant seats with type $\theta_\emptyset$ and lowest type of new teachers with type $\theta_0$ since there are none). In what follows, $f(x, y)$ will be the value of the statistic for the type value distribution $(x, y)$. Suppose the status-quo matching $\omega$ and set of profiles of teachers who do not participate in (re)assignment are as follows (where the letter name of the teacher also denotes the type of the teacher):

$$\begin{aligned}
\omega_{s_1} &= \{\ell_1\} && \text{and} && E^k_{s_1} = \{\ell^1_1, \ldots, \ell^{k-1}_1, \ell^k_1\},\\
\omega_{s_2} &= \{\ell_2\} && \text{and} && E^k_{s_2} = \{\ell^1_2, \ldots, \ell^{k-2}_2, h^{k-1}_2, h^k_2\},\\
\omega_{s_3} &= \{h_3\} && \text{and} && E^k_{s_3} = \{h^1_3, \ldots, h^{k-2}_3, \ell^{k-1}_3, \ell^k_3\},\\
\omega_{s_4} &= \{h_4\} && \text{and} && E^k_{s_4} = \{h^1_4, \ldots, h^{k-1}_4, h^k_4\};
\end{aligned}$$

such that $k \geq 3$ and for each school $s$, $\left|E^k_s\right| = k$. Note that this defines a sequence of economies for $k = 3, 4, \ldots$, which are not replica economies. We let $z^k_{s_i} := f(\delta^{\omega,k}_{s_i})$ for each $i \in \{1, \ldots, 4\}$. We use $\delta^{\omega*}_{s_i}$ to denote $\lim_{k\to\infty} \delta^{\omega,k}_{s_i}$. Note that $\delta^{\omega*}_{s_1} = \delta^{\omega*}_{s_2} = (1, 0)$ and $\delta^{\omega*}_{s_3} = \delta^{\omega*}_{s_4} = (0, 1)$. We let $z^*_{s_i} := \lim_{k\to\infty} f(\delta^{\omega,k}_{s_i}) = f(\delta^{\omega*}_{s_i})$ for each $i \in \{1, \ldots, 4\}$.

In the sequel, we assume that $v_h$ and $v_\ell$ are close enough to each other so that

$$3f(1,0) > f(0,1). \tag{20}$$

This is well-defined since $f$ is continuous and non-trivial.[77] With this, we can state and prove the following lemma.

**Lemma A.2.** *There is $\varepsilon > 0$ such that*

$$\frac{\partial \mathcal{G}}{\partial z_{s_1}}(z), \frac{\partial \mathcal{G}}{\partial z_{s_2}}(z) < 0 \textit{ and } \frac{\partial \mathcal{G}}{\partial z_{s_3}}(z), \frac{\partial \mathcal{G}}{\partial z_{s_4}}(z) > 0$$

*for each $z \in B_\varepsilon(z^*)$ where $\mathcal{G}$ is differentiable.*

**Proof.** Recall that $z^* = \big(f(1,0), f(1,0), f(0,1), f(0,1)\big)$. First, we show that

$$\frac{\partial \mathcal{G}}{\partial z_{s_1}}(z), \frac{\partial \mathcal{G}}{\partial z_{s_2}}(z) < 0$$

for any $z \in B_\varepsilon(z^*)$ where $\mathcal{G}$ is differentiable provided that $\varepsilon$ is small enough. We pick $\varepsilon > 0$ small enough so that any $z \in B_\varepsilon(z^*)$ satisfies $z_{s_1}, z_{s_2} < z_{s_3}, z_{s_4}$. Pick $z \in B_\varepsilon(z^*)$ and assume wlog that $z_{s_1} < z_{s_2} < z_{s_3} < z_{s_4}$ (the argument is the same for the other possible orderings consistent with $z_{s_1}, z_{s_2} < z_{s_3}, z_{s_4}$).[78] It is easily checked that

$$\frac{\partial \mathcal{G}}{\partial z_{s_2}}(z) < 0 \Longrightarrow \frac{\partial \mathcal{G}}{\partial z_{s_1}}(z) < 0,$$

and so we focus on proving that $\frac{\partial \mathcal{G}}{\partial z_{s_2}}(z) < 0$.

Given that the expression of Gini in our example is

$$\mathcal{G}(z) = \frac{-3z_{s_1} - z_{s_2} + z_{s_3} + 3z_{s_4}}{4(z_{s_1} + z_{s_2} + z_{s_3} + z_{s_1})},$$

Simple algebra yields that

$$\frac{\partial \mathcal{G}}{\partial z_{s_2}}(z) < 0$$

if and only if the following holds:

$$z_{s_1} - z_{s_3} - 2z_{s_4} < 0. \tag{21}$$

At $z^* = \big(f(1,0), f(1,0), f(0,1), f(0,1)\big)$ the above inequality simplifies to

$$f(1,0) < 3f(0,1),$$

[77] Indeed, $3f(1,0) > f(0,1)$ writes as $2f(1,0) > f(0,1) - f(1,0)$. As $v_h$ goes to $v_\ell > 0$, the right-hand side vanishes by continuity of $f$. The left-hand side remains strictly positive since $f$ is non-trivial.

[78] Recall that points where $\mathcal{G}$ is differentiable are those that are strictly ordered.

which is satisfied as $f(0,1) > f(1,0)$ through the fact that $f$ is strictly LD-increasing and as $f(0,1) > 0$ by non-triviality of $f$. Hence, Equation (21) holds at $z$ provided that $\varepsilon > 0$ is small enough. Thus, we obtain that for such $\varepsilon > 0$,

$$\frac{\partial \mathcal{G}}{\partial z_{s_1}}(z), \frac{\partial \mathcal{G}}{\partial z_{s_2}}(z) < 0$$

for any $z \in B_\varepsilon(z^*)$ where $\mathcal{G}$ is differentiable.

Now, we want to show that

$$\frac{\partial \mathcal{G}}{\partial z_{s_3}}(z), \frac{\partial \mathcal{G}}{\partial z_{s_4}}(z) > 0$$

for any $z \in B_\varepsilon(z^*)$ where $\mathcal{G}$ is differentiable provided here again that $\varepsilon > 0$ is small enough. Again, we pick $\varepsilon > 0$ small enough so that any $z \in B_\varepsilon(z^*)$ satisfies $z_{s_1}, z_{s_2} < z_{s_3}, z_{s_4}$. Pick $z \in B_\varepsilon(z^*)$ and assume wlog that $z_{s_1} < z_{s_2} < z_{s_3} < z_{s_4}$. Again, it is easily checked that

$$\frac{\partial \mathcal{G}}{\partial z_{s_3}}(z) > 0 \Longrightarrow \frac{\partial \mathcal{G}}{\partial z_{s_4}}(z) > 0.$$

Simple algebra yields,

$$\frac{\partial \mathcal{G}}{\partial z_{s_3}}(z) > 0$$

if and only if the following holds:

$$2z_{s_1} + z_{s_2} - z_{s_4} > 0. \tag{22}$$

At $z^* = \big(f(1,0), f(1,0), f(0,1), f(0,1)\big)$ the above inequality simplifies to

$$3f(1,0) > f(0,1),$$

which is satisfied by Equation(20). Hence, Equation (22) holds at $z$ provided that $\varepsilon > 0$ is small enough. Thus, we obtain that for such $\varepsilon > 0$,

$$\frac{\partial \mathcal{G}}{\partial z_{s_3}}(z), \frac{\partial \mathcal{G}}{\partial z_{s_4}}(z) < 0$$

for any $z \in B_\varepsilon(z^*)$ where $\mathcal{G}$ is differentiable. This completes the proof.

Recall that the relevant domain of $\mathcal{G}$ for the statistic $f$ in the market indexed by $k$ is

$$\mathcal{Z}^{f,k} = co\left\{f(\delta^{\mu,k}) : \mu \in \mathcal{M}\right\}.$$

In the sequel, we let $\varepsilon > 0$ be given by Lemma A.2. Note that for $k$ large enough the relevant domain $\mathcal{Z}^{f,k} \subset B_\varepsilon(z^*)$. Indeed, for any $\mu$, $\delta^{\mu,k} \to \delta^*$ and so, by continuity of $f$, we must have that $f(\delta^{\mu,k}) \to f(\delta^*) = z^*$. Hence, for $k$ large enough, $f(\delta^{\mu,k}) \in B_\varepsilon(z^*)$.

Given that there are finitely many possible matchings $\mu$, we obtain that for $k$ large enough, $f(\delta^{\mu,k}) \in B_\varepsilon(z^*)$ for each matching $\mu$. Hence, $\mathcal{Z}^{f,k} \subset B_\varepsilon(z^*)$.

Suppose the preference profile $P$ of the teachers who are participating in (re)assignment are given as follows:

$$\begin{aligned}
\ell_1 &: s_3 \; P_{\ell_1} \; s_1, \\
\ell_2 &: s_4 \; P_{\ell_2} \; s_2, \\
h_3 &: s_2 \; P_{h_3} \; s_1 \; P_{h_3} \; s_3, \\
h_4 &: s_2 \; P_{h_4} \; s_1 \; P_{h_4} \; s_4.
\end{aligned}$$

In this example, there are four individually rational matchings besides $\omega$:

$$\begin{array}{llll}
\mu^1_{s_1} = \{h_3\}, & \mu^1_{s_2} = \{h_4\}, & \mu^1_{s_3} = \{\ell_1\}, & \mu^1_{s_4} = \{\ell_2\}. \\
\mu^2_{s_1} = \{h_4\}, & \mu^2_{s_2} = \{h_3\}, & \mu^2_{s_3} = \{\ell_1\}, & \mu^2_{s_4} = \{\ell_2\}. \\
\mu^3_{s_1} = \{h_3\}, & \mu^3_{s_2} = \{\ell_2\}, & \mu^3_{s_3} = \{\ell_1\}, & \mu^1_{s_4} = \{h_4\}. \\
\mu^4_{s_1} = \{\ell_1\}, & \mu^4_{s_2} = \{h_4\}, & \mu^4_{s_3} = \{h_3\}, & \mu^4_{s_4} = \{\ell_2\}.
\end{array}$$

One can check that SI-CC yields matching $\mu^1$.[79] It should be clear that $\mu^1$ and $\mu^2$ are the matchings which yield the smallest value for the inequality index $\mathcal{G}$. More generally, let us prove that

$$\mathcal{G}\big(f(\delta^{\mu^1})\big) = \mathcal{G}\big(f(\delta^{\mu^2})\big) < \mathcal{G}\big(f(\delta^{\mu^3})\big), \mathcal{G}\big(f(\delta^{\mu^4})\big) < \mathcal{G}\big(f(\delta^{\omega})\big). \tag{23}$$

The equality is straightforward. As for the inequalities, let us first start with the strict inequality, i.e.,

$$\mathcal{G}\big(f(\delta^{\mu^1})\big) < \mathcal{G}\big(f(\delta^{\mu^3})\big).$$

Consider changing the matching from $\mu^3$ to $\mu^1$. We denote the change in the value of the statistic at school $s$ by $\Delta_s$. First, the value of the statistic does not vary for $s_1$ and $s_3$. For school $s_2$, it varies by $\Delta_{s_2} = f\left(\frac{k-3}{k}, \frac{3}{k}\right) - f\left(\frac{k-2}{k}, \frac{2}{k}\right) > 0$ and for school $s_4$ it varies by $\Delta_{s_4} = f\left(\frac{1}{k}, \frac{k-1}{k}\right) - f(0,1) < 0$ (the strict inequalities use our assumption that $f$ is strictly LD-increasing).

Since, by construction, $\mathcal{G}$ is differentiable on its relevant domain $\mathcal{Z}^{f,k}$, its total differential satisfies $d\mathcal{G} = \sum_{s\in S} \frac{\partial \mathcal{G}}{\partial z_s}(z)dz_s$.[80] To calculate the total variation in the inequality index from $\mu^3$ to $\mu^1$, we obtain the following path integral on the linear path in variable $u \in [0,1]$

[79] In the sequel, SI-CC refers to SI-CC defined with respect to the natural type ranking.

[80] This comes from the fact that, by construction, at any matching $\mu$, the value of the statistic at $s_i$ is always strictly smaller than the value of the statistics at $s_{i+1}$ for each $i$. So at any point $z$ in the relevant domain, there is no pair $s, s'$ such that $z_s = z_{s'}$. Hence, $\mathcal{G}$ is indeed differentiable on its relevant domain.

so that $\hat{z}_s(u) = u\Delta_s + f(\delta_s^{\mu^3})$ for each school $s$, and $\hat{z}(u) = \big(\hat{z}_s(u)\big)_{s\in S}$. We obtain

$$\begin{aligned}
\mathcal{G}\big(f(\delta^{\mu^1})\big) - \mathcal{G}\big(f(\delta^{\mu^3})\big) &= \int_{\mathcal{G}\left(f(\delta^{\mu^3})\right)}^{\mathcal{G}\left(f(\delta^{\mu^1})\right)} d\mathcal{G} = \int_0^1 \sum_{s\in S} \Delta_s \frac{\partial \mathcal{G}}{\partial z_s}\big(\hat{z}(u)\big)du \\
&= \int_0^1 \left[\Delta_{s_2}\frac{\partial \mathcal{G}}{\partial z_{s_2}}\big(\hat{z}(u)\big) + \Delta_{s_4}\frac{\partial \mathcal{G}}{\partial z_{s_4}}\big(\hat{z}(u)\big)\right] du \\
&< 0,
\end{aligned}$$

where the strict inequality uses the fact that

$$\frac{\partial \mathcal{G}}{\partial z_{s_2}}\big(\hat{z}(u)\big) < 0, \frac{\partial \mathcal{G}}{\partial z_{s_4}}\big(\hat{z}(u)\big) > 0$$

for each $u \in [0,1)$. This comes from the fact that $\hat{z}(u) \in \mathcal{Z}^{f,k} \subset B_\varepsilon(z^*)$. Hence, we must have

$$\mathcal{G}\big(f(\delta^{\mu^1})\big) < \mathcal{G}\big(f(\delta^{\mu^3})\big).$$

Note that a same argument applies to show that $\mathcal{G}\big(f(\delta^{\mu^1})\big) < \mathcal{G}\big(f(\delta^{\mu^4})\big)$.

Now, to complete the proof for Equation (23), let us show the weak inequality, i.e.,

$$\mathcal{G}\big(f(\delta^{\mu^4})\big) < \mathcal{G}\big(f(\delta^{\omega})\big)$$

(here again, the same argument applies to prove that $\mathcal{G}\big(f(\delta^{\mu^3})\big) < \mathcal{G}\big(f(\delta^{\omega})\big)$).

Consider changing the matching from $\omega$ to $\mu^4$. Again, let us denote the change in the value of the statistic at school $s$ by $\Delta_s$. First, the value of the statistic does not vary for $s_1$ and $s_3$. For school $s_2$, it varies by $\Delta_{s_2} = f(\frac{k-3}{k}, \frac{3}{k}) - f(\frac{k-2}{k}, \frac{2}{k}) > 0$ and for school $s_4$ it varies by $\Delta_{s_4} = f(\frac{1}{k}, \frac{k-1}{k}) - f(0,1) < 0$ (the strict inequalities use our assumption that $f$ is strictly LD-increasing). By a similar argument as above, we have that the total variation in the inequality index from $\omega$ to $\mu^4$ corresponds to the following path integral on the linear path in variable $u \in [0,1]$ so that $\bar{z}_s(u) = u\Delta_s + f(\delta_s^{\omega})$ for each school $s$, and $\bar{z}(u) = \big(\bar{z}_s(u)\big)_{s\in S}$,

$$\begin{aligned}
\mathcal{G}\big(f(\delta^{\mu^4})\big) - \mathcal{G}\big(f(\delta^{\omega})\big) &= \int_{\mathcal{G}\left(f(\delta^{\omega})\right)}^{\mathcal{G}\left(f(\delta^{\mu^4})\right)} d\mathcal{G} = \int_0^1 \sum_{s\in S} \Delta_s \frac{\partial \mathcal{G}}{\partial z_s}\big(\bar{z}(u)\big)du \\
&= \int_0^1 \left[\Delta_{s_2}\frac{\partial \mathcal{G}}{\partial z_{s_2}}\big(\bar{z}(u)\big) + \Delta_{s_4}\frac{\partial \mathcal{G}}{\partial z_{s_4}}\big(\bar{z}(u)\big)\right] du \\
&< 0.
\end{aligned}$$

where the strict inequality uses the fact that

$$\frac{\partial \mathcal{G}}{\partial z_{s_2}}\big(\bar{z}(u)\big) < 0, \quad \frac{\partial \mathcal{G}}{\partial z_{s_4}}\big(\bar{z}(u)\big) > 0$$

for each $u \in [0, 1)$. Again, this comes from the fact that $\bar{z}(u) \in \mathcal{Z}^{f,k} \subset B_\varepsilon(z^*)$. Hence, Equation (23) holds true.

Now, let us consider a mechanism $\varphi$ which is individually rational, strategy-proof and has less inequality than SI-CC whenever possible. Because at preference profile $P$, SI-CC selects $\mu^1$, and $\varphi$ has less inequality than SI-CC, by Equation (23), $\varphi$ must select either $\mu^1$ or $\mu^2$ at $P$. First, suppose $\varphi(P) = \mu^1$. When $h_3$ submits an alternative preference relation deeming $s_1$ unacceptable,

$$h_3 : s_2 \; P'_{h_3} \; s_3,$$

she will be better off. To see this, observe that $h_3$ will no longer be matched with $s_1$ in an individually rational matching for the new preference profile $(P'_{h_3}, P_{-h_3})$. Now, under SI-CC, one can check that $h_3$ remains at $s_3$ and the matching achieved is $\mu^4$. Further, by Equation (23), the unique individually rational matching with strictly less inequality than $\mu^4$ is matching $\mu^2$. Therefore, we should have

$$\varphi\left(P'_{h_3}, P_{-h_3}\right) = \mu^2.$$

Thus, teacher $h_3$ receives her first choice post at $s_2$ by omitting $s_1$ from her list, and she successfully manipulates $\varphi$ if $\varphi(P) = \mu^1$, a contradiction with the strategy-proofness of $\varphi$.

Second, suppose $\varphi(P) = \mu^2$. When $h_4$ submits an alternative preference relation deeming $s_1$ unacceptable,

$$h_4 : s_2 \; P'_{h_4} \; s_4,$$

she will be better off. To see this, observe that $h_4$ will no longer be matched with $s_1$ in an individually rational matching for the new preference profile $(P'_{h_4}, P_{-h_4})$. Now, under SI-CC, one can check that the selected matching remains $\mu^1$. Further, by Equation (23), all other individually rational matchings have strictly more inequality than $\mu^1$. Therefore, we should have

$$\varphi\left(P'_{h_4}, P_{-h_4}\right) = \mu^1.$$

Thus, teacher $h_4$ receives her first choice post at $s_2$ by omitting $s_1$ from her list, and she successfully manipulates $\varphi$ if $\varphi(P) = \mu^2$, a contradiction with the strategy-proofness of $\varphi$.

**Remark 1.** *The argument can be applied beyond the Gini index. For instance, it applies to the T20/B20 ratio inequality index as defined in Section 4. In the context of the example, this is*

$$\mathcal{I}(z) = \frac{\max\{z_{s_1}, z_{s_2}, z_{s_3}, z_{s_4}\}}{\min\{z_{s_1}, z_{s_2}, z_{s_3}, z_{s_4}\}}.$$

*Note that, as for the Gini index, this index is differentiable at points that are strictly ordered. In* *addition, a version of Lemma A.2 holds in the sense that there is* $\varepsilon > 0$ *such that*

$$\frac{\partial \mathcal{I}}{\partial z_{s_1}}(z) \leq 0, \quad \frac{\partial \mathcal{I}}{\partial z_{s_2}}(z) \leq 0$$

*for each* $z \in B_\varepsilon(z^*)$ *where* $\mathcal{I}$ *is differentiable. In addition, the inequality is strict for* $\{z_{s_i}\} = \arg\min\{z_{s_1}, z_{s_2}\}$ *and, similarly,*

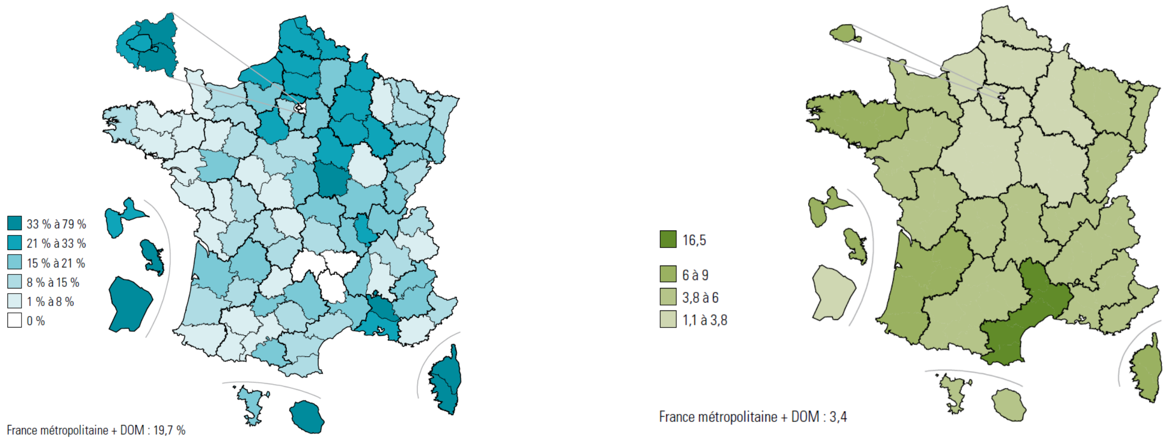
$$\frac{\partial \mathcal{I}}{\partial z_{s_3}}(z) \geq 0, \quad \frac{\partial \mathcal{I}}{\partial z_{s_4}}(z) \geq 0$$

*for each* $z \in B_\varepsilon(z^*)$ *where* $\mathcal{I}$ *is differentiable. Here again, the inequality is strict for* $\{z_{s_i}\} = \arg\max\{z_{s_3}, z_{s_4}\}$*. Given this, one can check that the argument goes through as well for this inequality index. More generally, if we have an inequality index* $\mathcal{I}$ *which is differentiable at points that are strictly ordered and the above version of Lemma A.2 holds then Proposition A.4 holds for this inequality index as well.*

# B Other Figures and Tables

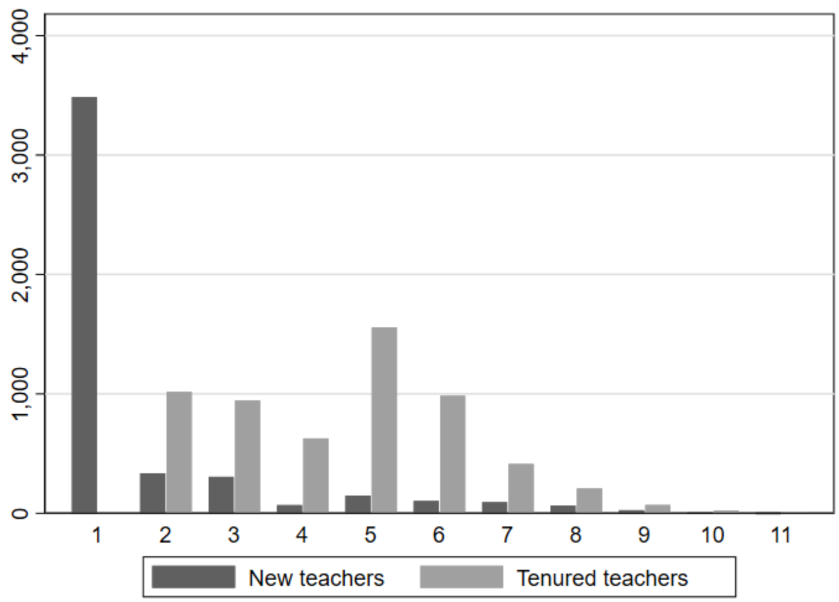
**Figure A.1:** Share of Disadvantaged Students and Older to Younger Teacher Ratio in France

*Share of students in a priority education school* | *Ratio of teachers age 50+ to age 30–*




Notes: The left map plots the share of students enrolled in a "priority education" school in each region, a label given to the most disadvantaged schools in France. The right map plots the ratio of the number of teachers older than 50 to the number of teachers younger than 30. Column (3) of Table A.4 provides the underlying statistics. Age can be used as a proxy for the experience level of a teacher. The ratio is equal to 1.1 and 1.6 in Créteil and Versailles, respectively. In contrast, the most attractive region, Rennes, had almost 7.4 and 5 times more teachers older than 50 than teachers younger than 30 compared to Créteil and Versailles, respectively.

**Figure A.2:** Distribution of Teacher Experience Types




Notes: This figure shows the number of teachers with each experience type. We classify teachers into eleven experience bins with each bin referring to a separate type.

**Figure A.3:** Definition of Teacher Experience Types

| **Type** | **Vacant position** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Min experience (in years) | 0 | 0 | 1 | 2 | 3 | 4 | 5 | 9 | 13 | 17 | 21 | 25 |
| Max experience (in years) | 0 | 0 | 1 | 2 | 3 | 4 | 8 | 12 | 16 | 20 | 24 | 28 |
| Type value | 0 | 0 | 1 | 2 | 3 | 4 | 6.5 | 10.5 | 14.5 | 18.5 | 22.5 | 26.5 |

Notes: This figure describes, for each of the 11 teacher experience types, the minimum experience of the type, its maximum experience, and its mean experience (noted type value). We consider vacant positions as a separate type with a mean experience of zero. Each new cohort starts their job in September. For example, if a teacher started her tenured job in September 2005 after the centralized match, she would appear with the experience level of 8 years in 2013 in our data set and be designated to type 6.

**Table A.1:** Descriptive Statistics on Teachers and Regions

| | Tenured Teachers | | | New Teachers | | |
|---|---|---|---|---|---|---|
| | French | Math | English | French | Math | English |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Panel A. Characteristics of teachers | | | | | | |
| Female (%) | 78.8 | 48.9 | 84.3 | 80.7 | 44.4 | 82.7 |
| Married (%) | 47.6 | 44.1 | 44.7 | 41.3 | 42.4 | 43.9 |
| Is in priority education school (%) | 14.1 | 12.6 | 8.1 | - | - | - |
| Experience (yrs) | 6.69 | 5.31 | 5.64 | 1.76 | 1.16 | 1.02 |
| Has advanced teaching qualif. (%) | 15.9 | 20.7 | 7.3 | 13.6 | 30.6 | 12.9 |
| Panel B. Characteristics of the regions to which teachers are assigned at status-quo | | | | | | |
| Is the teacher's birth region (%) | 12.9 | 10.7 | 11.8 | - | - | - |
| Is Créteil or Versailles (%) | 53.0 | 56.2 | 46.9 | - | - | - |
| Is in South of France (%) | 7.1 | 7.1 | 7.8 | - | - | - |
| Students in urban areas (%) | 66.4 | 68.9 | 66.1 | - | - | - |
| Disadvantaged students (%) | 53.6 | 53.9 | 53.6 | - | - | - |
| Students in priority education (%) | 25.4 | 25.4 | 24.2 | - | - | - |
| Students in private school (%) | 17.7 | 17.7 | 18.5 | - | - | - |
| Teachers younger than 30 yrs (%) | 13.5 | 13.5 | 12.5 | - | - | - |
| Panel C. Characteristics of the regions teachers rank first | | | | | | |
| Is the teacher's birth region (%) | 35.8 | 46.1 | 42.6 | 35.3 | 39.7 | 40.3 |
| Is in South of France (%) | 22.1 | 26.6 | 30.7 | 17.1 | 18.4 | 20.5 |
| Is Créteil or Versailles (%) | 4.4 | 3.8 | 3.2 | 1.4 | 1.5 | 1.4 |
| Students in urban area (%) | 66.7 | 52.4 | 51.4 | 62.8 | 59.5 | 60.5 |
| Disadvantaged students (%) | 52.8 | 53.3 | 53.7 | 53.1 | 53.3 | 53.0 |
| Students in priority education (%) | 20.3 | 16.5 | 15.4 | 21.6 | 20.5 | 20.2 |
| Students in private school (%) | 26.4 | 25.1 | 26.5 | 22.7 | 22.0 | 22.5 |
| Teachers younger than 30 yrs (%) | 6.5 | 6.4 | 6.0 | 8.2 | 8.6 | 8.3 |
| Observations (#) | 859 | 605 | 629 | 786 | 958 | 750 |

Notes: This table reports descriptive statistics for teachers and regions in three subjects: French, Math, and English. Statistics are reported for the sample of teachers we use for the demand estimations. Columns (1) to (3) report statistics for tenured teachers. Columns (4) to (6) report statistics for new teachers. New teachers have missing values for statistics related to the region of status-quo assignment. We discard teachers who submit a joint list with their partner, teachers who are from one of the six regions that are overseas, and teachers for whom one of the individual characteristics is missing. The last row reports the number of teachers in each subject. Panels A, B, and C, respectively, present descriptive statistics of teachers, of the region to which they are assigned at the status quo, and of the region they rank first. Appendix J provides a detailed description of each variable.

**Table A.2:** Descriptive Statistics on Region Types

| | Share of subjects in which region is low type | Mean experience across subjects (in years) | Share of teachers with fewer than 4 years of experience | Region weight |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| Créteil | 100% | 10.05 | 22.6% | 7.6% |
| Versailles | 100% | 11.15 | 19.1% | 9.9% |
| Amiens | 100% | 11.71 | 16.5% | 3.3% |
| Orléans-Tours | 100% | 12.99 | 10.7% | 4.2% |
| Nice | 87.5% | 13.15 | 9.4% | 3.4% |
| Lyon | 87.5% | 13.27 | 7.5% | 4.7% |
| Dijon | 75% | 13.23 | 13.2% | 2.7% |
| Toulouse | 75% | 13.63 | 6.9% | 4.8% |
| Grenoble | 62.5% | 13.50 | 9.9% | 5.3% |
| Rouen | 62.5% | 13.53 | 8.9% | 3.3% |
| Strasbourg | 62.5% | 13.72 | 9.0% | 2.8% |
| Lille | 50% | 13.41 | 9.4% | 6.5% |
| Aix-Marseille | 50% | 13.61 | 7.6% | 4.7% |
| Limoges | 50% | 13.70 | 7.2% | 1.2% |
| Reims | 37.5% | 13.83 | 8.7% | 2.1% |
| Paris | 37.5% | 14.26 | 10.3% | 2.9% |
| Montpellier | 25% | 13.93 | 7.1% | 4.5% |
| Besancon | 25% | 13.97 | 8.9% | 2.0% |
| Poitiers | 25% | 14.06 | 7.1% | 2.7% |
| Caen | 12.5% | 13.93 | 6.9% | 2.3% |
| Nantes | 12.5% | 14.01 | 7.7% | 4.5% |
| Clermont-Ferrand | 12.5% | 14.08 | 6.7% | 1.9% |
| Nancy-Metz | 0% | 14.49 | 5.4% | 3.6% |
| Bordeaux | 0% | 14.51 | 6.4% | 5.0% |
| Rennes | 0% | 15.00 | 5.5% | 3.9% |

Notes: This table reports descriptive statistics on region types and characteristics. Region types are defined separately for each subject. We use the sign of the partial derivatives of the T20/B20 ratio index to create two groups of regions: (1) Low-type regions have a negative derivative (or a zero derivative but an average experience at the status quo that is lower than the median experience), and (2) high-type regions have a positive derivative (or a zero derivative but an average experience at the status quo that is higher than the median experience). Differences between the distribution of teacher experience across regions in different subjects mean that a region can be low-type in one subject but high-type in another. The first column reports the share of subjects in which a region is classified as low-type. Columns 2 and 3 report the average teacher experience at the status quo and the share of teachers with fewer than four years of experience, respectively. Column 4 reports the region weight, defined as the region's relative size. The size captures the number of teachers in the region and the number of vacant positions.

**Table A.3:** Number of teachers and vacant positions

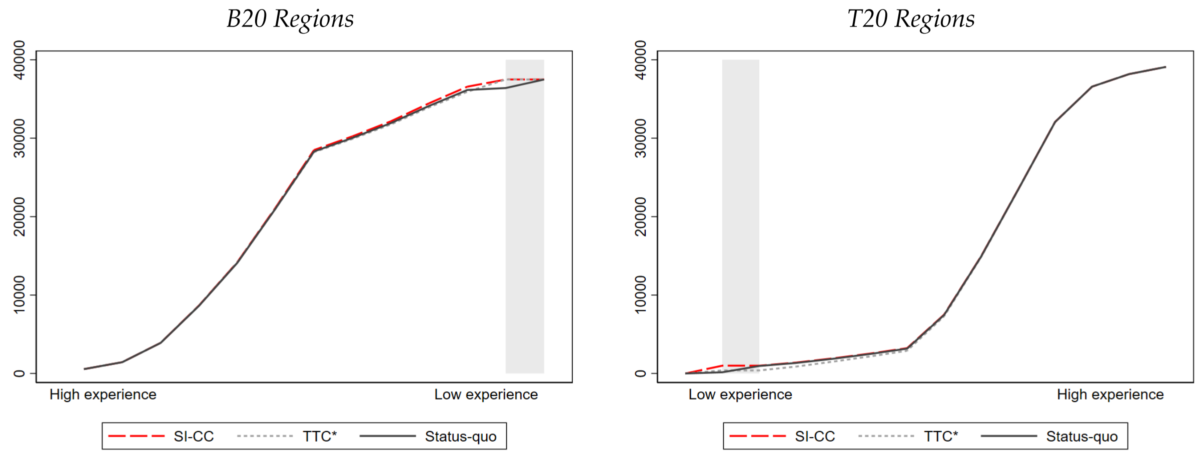
| Subjects | All teachers (1) | New teachers (2) | Tenured teachers (3) | Vacant positions (4) |
|---|---|---|---|---|
| All subjects | 10,483 | 4,637 | 5,846 | 3,912 |
| Sports | 2,072 | 569 | 1,503 | 475 |
| French | 1,645 | 786 | 859 | 663 |
| Math | 1,563 | 958 | 605 | 824 |
| English | 1,379 | 750 | 629 | 646 |
| History-Geography | 1,235 | 660 | 575 | 567 |
| Spanish | 1,003 | 317 | 686 | 249 |
| Physics-Chemistry | 838 | 311 | 527 | 254 |
| Biology | 748 | 286 | 462 | 246 |

**Table A.4:** Statistics on Regions

| Regions | Ratio: # of tenured teachers asking to enter / exit the region (1) | % of teachers asking for a new assignment coming from the region (2) | Ratio: # of teachers aged more than 50 / less than 30 (3) | % of students enrolled in priority education (4) | % of students whose reference parent has no diploma (5) | % of students obtaining their baccalaureate (6) |
|---|---|---|---|---|---|---|
| Rennes | 15.55 | 0.5 | 8.10 | 7.9 | 14.18 | 91.54 |
| Bordeaux | 8.95 | 0.8 | 6.56 | 14.6 | 19.22 | 86.25 |
| Toulouse | 6.56 | 1.5 | 5.29 | 8.9 | 17.38 | 88.57 |
| Paris | 3.02 | 2.8 | 6.90 | 25.5 | 21.54 | 85.48 |
| Aix-Marseille | 2.54 | 1.9 | 5.08 | 30.1 | 27.20 | 81.77 |
| Grenoble | 1.74 | 2.3 | 3.91 | 16.5 | 19.80 | 88.17 |
| Amiens | 0.08 | 6.2 | 1.89 | 23.9 | 27.71 | 82.41 |
| Créteil | 0.03 | 22.7 | 1.14 | 35.5 | 31.62 | 83.94 |
| Versailles | 0.05 | 25.7 | 1.62 | 24.9 | 21.88 | 87.92 |

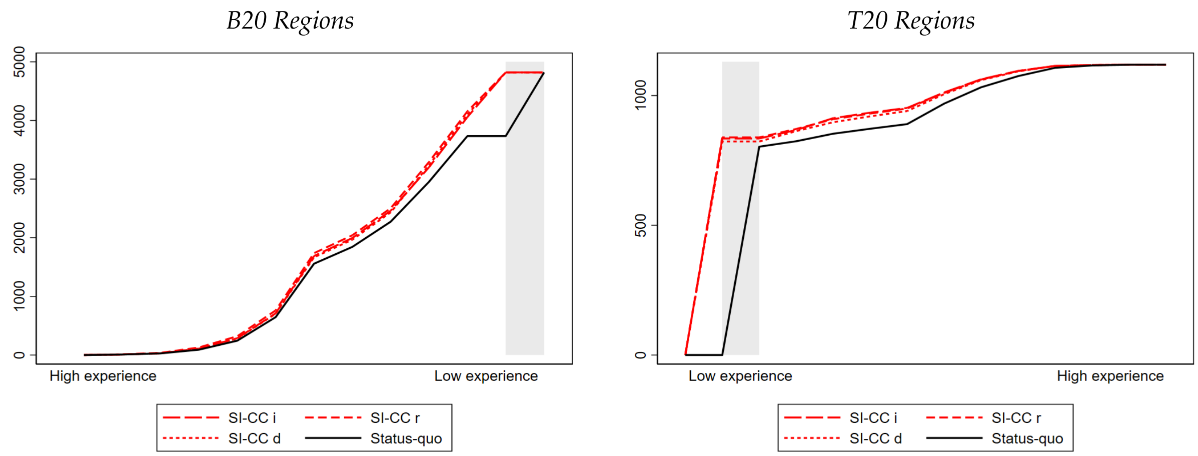
Notes: This table reports descriptive statistics for the three most attractive regions (Rennes, Bordeaux, and Toulouse), the three least attractive regions (Amiens, Créteil, and Versailles), and three intermediate regions (Paris, Aix-Marseille, and Grenoble). Attractiveness is measured by the ratio of the number of tenured teachers asking to enter a region to the number of teachers asking to leave the region (reported in column 1). All statistics reported in this table come from the following reference: Direction de l'Evaluation de la Prospective et de la Performance (2014). In column (1), the number of teachers asking to enter the region corresponds to the number of teachers who rank the region as their first choice in their preference list, while the number of teachers asking to leave the region corresponds to the number of teachers who are initially assigned the region and submit a preference list to move to another region.

**Figure A.4:** Cumulative Distribution of Teacher Experience Types in the Entire Market



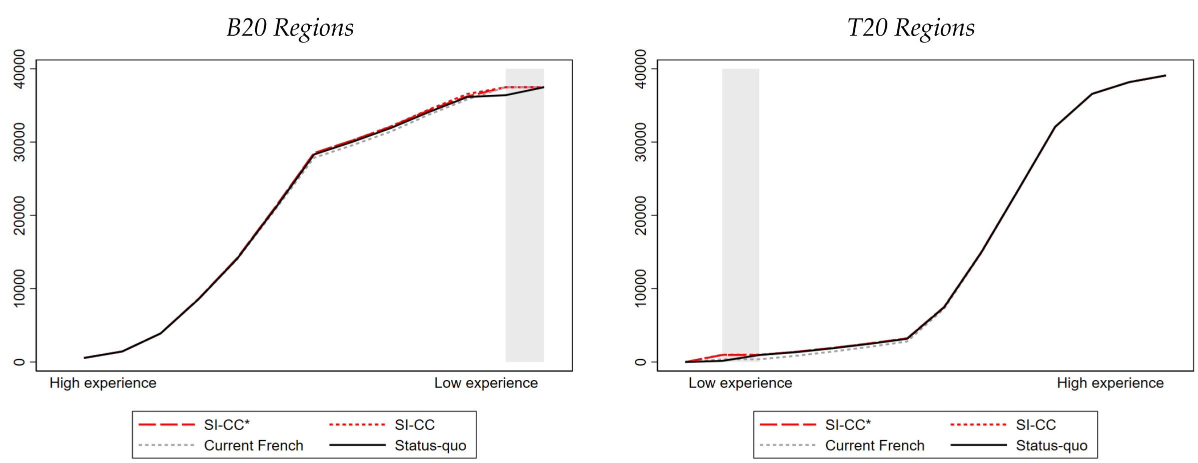

Notes: This figure shows the cumulative distribution of teacher experience types in the entire market, i.e., including participating and non-participating teachers, in the B20 regions (left) and in T20 regions (right) under SI-CC, TTC*, and status quo. We identify T20 and B20 regions in each subject and find the cumulative distribution aggregated across subjects. The horizontal axis reports the eleven experience types of teachers, ordered from most experienced to least experienced (left panel) and from least experienced to most experienced (right panel) in accordance with the natural type rankings of these regions. The area shaded in gray corresponds to vacant positions.

**Figure A.5:** Cumulative Distribution of Teacher Experience Types for the (Re)assignment Market - Different Chain Selection Rules




Notes: This figure shows the cumulative distribution of teacher experience types in B20 and T20 regions for the (re)assignment market using three different chain selection rules under SI-CC. The suffixes “i”, “r”, and “d” respectively stand for increasing, random, and decreasing. These orderings mean that the teachers starting a chain are respectively selected by increasing, random, and decreasing order of their maximum Ministry-mandated priority points. The left panel reports the distribution in the B20 regions of France, and the right panel the distribution in the T20 regions of France. The horizontal axis reports the eleven experience types of teachers, ordered from most experienced to least experienced (left panel) and from least experienced to most experienced (right panel). The area shaded in gray corresponds to vacant positions. The thick dark gray line (marked as “Status-quo” in the legend) corresponds to the cumulative distribution of teacher types at the status-quo matching.

**Figure A.6:** Cumulative Distribution of Teacher Experience in the Entire Market - SI-CC* vs. SI-CC and Current French




Notes: See the caption of Figure A.4 for the construction methodology of this figure.

**Figure A.7:** Change in Region Experience (in years) subject by subject - (Re)assignment market

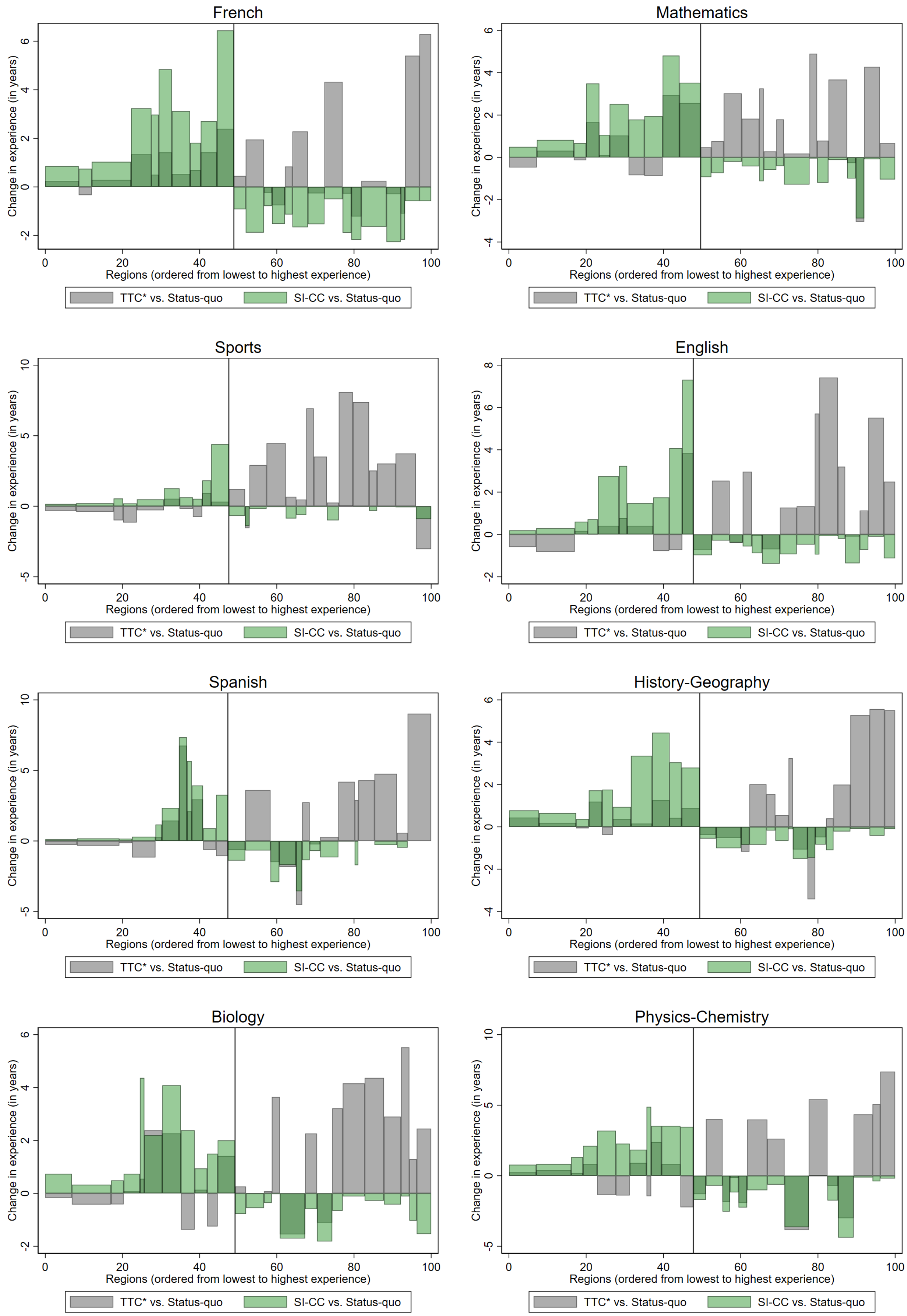
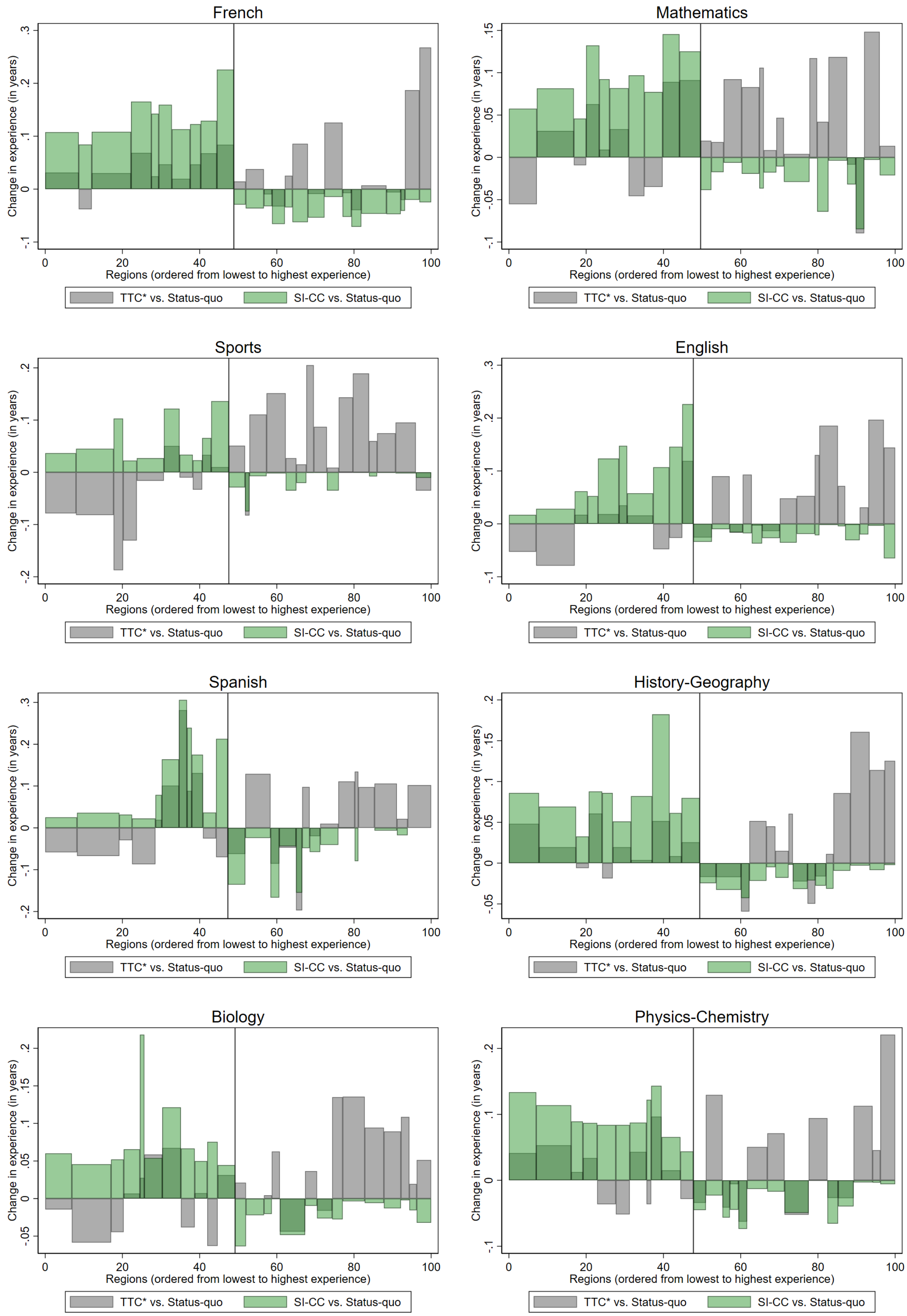


**Figure A.8:** Change in Region Experience (in years) subject by subject - Entire market



# C Other Applications

In this appendix, we explain other applications of our mechanisms in more detail. We give three concrete applications.

**Intra-district school choice after a status-quo assignment.** In the US, Austin Independent School District (AISD) of Texas assigns students to schools through an address based matching procedure.[81] Unfortunately, address based assignment ends up with segregated schools.[82] In order to eliminate the segregation and fill the empty seats at the under-demanded schools, AISD runs a transfer procedure in which a student who is in relative demographic majority in her assigned school can apply to the schools in which she belongs to the minority demographic group. Moreover, new students who arrive at the district after the matching procedure is run can also participate in this transfer procedure.[83]

In addition to the transfer programs to achieve racial diversity at schools, many school districts, including Davenport, IA (DCS, 2019), and Seminole, FL (SCPS, 2021), run transfer programs to achieve diversity in terms of the socioeconomic status (SES) of the students. Diversity transfer programs based on SES are also suggested to the school district by The United States Department of Education and the United States Department of Justice (ED, 2011).

**Job rotation.** Job rotation is defined as the horizontal movement of employees among different positions in a company. It is a well-established and commonly practiced human resource management program in many settings. It benefits companies through employee enrichment and success of developing future managers as well as decreased worker turnover due to increased job satisfaction of the participants (Cheraskin and Campion, 1996).

Job rotation programs can also be used as a means to obtain certain distributional goals of a company such as achieving gender balance across different departments of the company and retention of female employees and increasing the development of more female leaders through rotation programs.[84]

**Other civil services.** There are other centralized matching procedures for civil servants from different professions. For example, police officers are assigned to neighborhoods by centralized procedures in several US cities such as Chicago (Sidibe et al., 2021); doctors are assigned to government hospitals in some countries such as Turkey and for their first

[81]There are 128 school programs in AISD. In the 2020-2021 school year, the total enrolment in AISD is more than 75,000 (AISD, 2021).

[82]In 2019, the student body at 15% and 63% of the elementary schools were composed of more than 60% white and hispanic-black students, respectively.

[83]The majority minority transfer program is used in many school districts in the US including Huntsville, AL (HCS, 2020), Suffolk, VA (SPSK12, 2021), and Florence, SC (F1S, 2019).

[84]Observe that companies use professionally designed centralized matching software for job rotations, for example see `https://www.tws-partners.com/corporate-functions/hr/`.

residency jobs in many countries including Canada, the U.K., and the U.S. (Roth, 1984); civil administrators are assigned centrally for example in India (Thakur, 2020).

In police officer (re)assignment, senior officers are known to shy away from urban centers, while the police departments would like more of officers in urban centers due to disproportionate crime rates, similar to our distributional problems in teacher assignment.

Another concrete example in this domain is the Indian Administrative Services, the top-tier government jobs in India. This selective service conducts first time assignment of officials to regional government jobs in states of India every year, while reassignment is conducted separately. The state has distributional objectives based on spread of talent across states, constitutional affirmative action, and respect of preferences for home states. Our procedures can be used in these domains as well to achieve different distributional objectives.

# D Examples

We illustrate how SI-CC works with the following example:

**Example A.1.** *Let* $S = \{s_1, s_2, s_3, s_4\}$, $T = \{h_1, \ell_1, m_2, h_3, m_3, o_N\}$. *The status-quo matching* $\omega$ *is given as*

$$\omega_{s_1} = \{h_1, \ell_1\}, \;\; \omega_{s_2} = \{m_2\}, \;\; \omega_{s_3} = \{h_3, m_3\}, \;\; \omega_{s_4} = \emptyset,$$

*and* $o_N$ *is a new teacher. Let* $q_{s_1} = q_{s_3} = 2$ *and* $q_{s_2} = q_{s_4} = 1$. *The preferences of teachers are:*

$$\begin{aligned}
& s_2 \; P_{h_1} \; s_4 \; P_{h_1} \; s_3 \; P_{h_1} \; s_1 \; P_{h_1} \; \emptyset \\
& s_3 \; P_{\ell_1} \; s_1 \; P_{\ell_1} \; s_4 \; P_{\ell_1} \; \emptyset \; P_{\ell_1} \; s_2 \\
& s_4 \; P_{m_2} \; s_3 \; P_{m_2} \; s_2 \; P_{m_2} \; s_1 \; P_{m_2} \; \emptyset \\
& s_1 \; P_{h_3} \; s_3 \; P_{h_3} \; s_2 \; P_{h_3} \; s_4 \; P_{h_3} \; \emptyset \\
& s_2 \; P_{m_3} \; s_1 \; P_{m_3} \; s_3 \; P_{m_3} \; s_4 \; P_{m_3} \; \emptyset \\
& s_2 \; P_{o_N} \; s_1 \; P_{o_N} \; s_3 \; P_{o_N} \; s_4 \; P_{o_N} \; \emptyset
\end{aligned}$$

*Let* $\Theta = \{h, m, \ell, o\}$, *and the letter name of each teacher is her type.*[85] *The type ranking profile* $\trianglerighteq$ *is given as, observing* $\theta_\emptyset$ *is the vacant seat type and is redundant to rank for* $s_1$, $s_2$, *and* $s_3$ *which*

[85] Type $o$ is the "no experience" new teacher type.

*have no vacant seats:*[86]

$$
\begin{aligned}
&h \rhd_{s_1} m \rhd_{s_1} \ell \rhd_{s_1} o \rhd_{s_1} \theta_{\emptyset}\\
&o \rhd_{s_2} \theta_{\emptyset} \rhd_{s_2} \ell \rhd_{s_2} m \rhd_{s_2} h\\
&o \rhd_{s_3} \theta_{\emptyset} \rhd_{s_3} \ell \rhd_{s_3} m \rhd_{s_3} h\\
&h \rhd_{s_4} m \rhd_{s_4} \ell \rhd_{s_4} o \rhd_{s_4} \theta_{\emptyset}
\end{aligned}
$$

*In Step 1 of SI-CC, we obtain the graph in Figure A.9 (such that the status-quo employees of each school are placed in a dashed bubble around the school). Notice that, $h_1$ does not point to her top choice, $s_2$, since neither improvement condition holds for $s_2$ via her. There exists a cycle, $(h_3, s_1, \ell_1, s_3)$, in which every school satisfies Improvement Condition 1. We execute that cycle by assigning $h_3$ and $\ell_1$ to $s_1$ and $s_3$, respectively.*

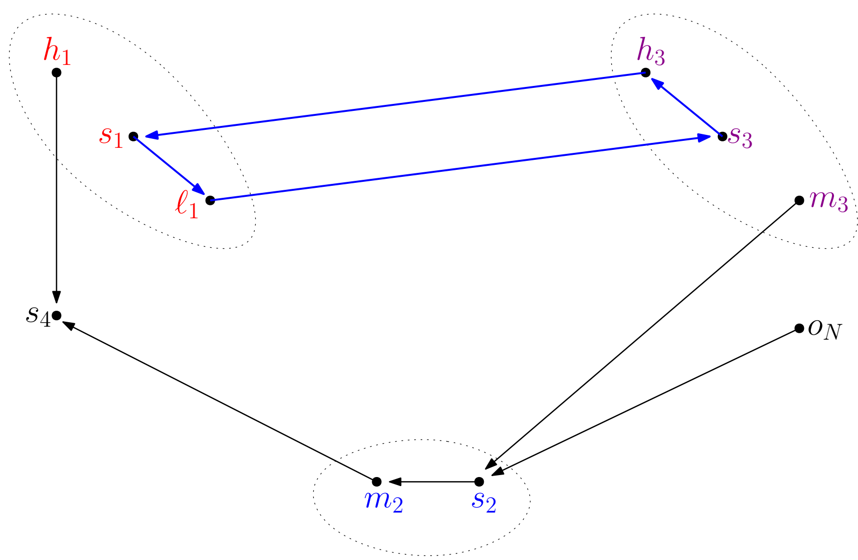



**Figure A.9:** Graph of Step 1 of SI-CC

*In Step 2 of SI-CC, we obtain the graph in Figure A.10. There exists no cycle. We execute chain $(o_N, s_2, m_2, s_4)$, whose first teacher is the only new teacher $o_N$, by assigning $o_N$ and $m_2$ to $s_2$ and $s_4$, respectively.*

[86] In general, the ranking of the vacant seat type matters for ensuring a reduction of inequality as detailed in Section 4. Also note that since there is a single new teacher and no two status-quo employees of a school are of the same type (given strict type rankings), no tiebreaker is needed in this example.

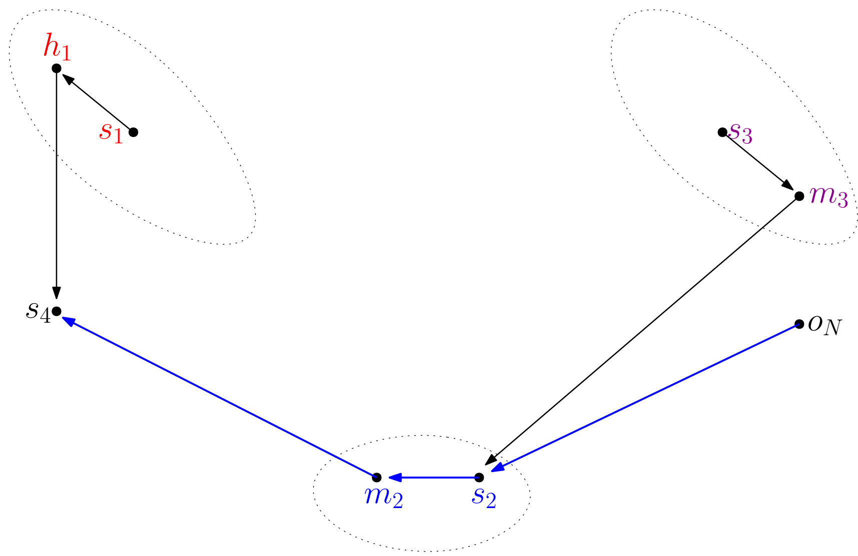



**Figure A.10:** Graph of Step 2 of SI-CC

*In Step 3 of SI-CC, we obtain the graph in Figure A.11. Notice that, $h_1$ ($m_3$) points to $s_3$ ($s_1$) even though she has a worse type than the teacher pointed by $s_3$ ($s_1$). Such a situation is possible due to the positive balance buffer achieved as a result of the exchanges executed in the earlier steps. There exists a cycle, $(h_1, s_3, m_3, s_1)$, in which every school satisfies Improvement Condition 1. We execute that cycle by assigning $h_1$ and $m_3$ to $s_3$ and $s_1$, respectively.*

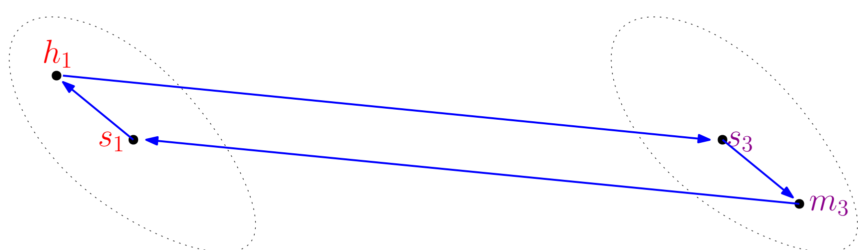



**Figure A.11:** Graph of Step 3 of SI-CC

*The outcome matching of SI-CC is $\mu$ such that*

$$\mu_{s_1} = \{h_3, m_3\}, \;\; \mu_{s_2} = \{o_N\}, \;\; \mu_{s_3} = \{\ell_1, h_1\}, \;\; \mu_{s_4} = \{m_2\}.$$

Example A.2 below shows that SI-CC can be manipulated by a teacher and it is not constrained-efficient under an alternative pointing rule.

**Example A.2.** *Let $S = \{s, s', s''\}$, $T = \{t_1, t_2, t_3, t_4\}$, the status-quo matching be*

$$\omega_s = \{t_1, t_2\}, \omega_{s'} = \{t_3\}, \omega_{s''} = \{t_4\},$$

*and $q_s = 2$, $q_{s'} = q_{s''} = 1$. Notice that, there is no vacant seat in any school. Teachers $t_2$, $t_3$, and $t_4$ have the same types, i.e., $\tau(t_2) = \tau(t_3) = \tau(t_4)$. The type ranking profile $\trianglerighteq$ is given as:*

*$\tau(t_1) \rhd_s \tau(t)$ for all $t \neq t_1$. The preferences of the teachers are*

$$sP_{t_1}s'P_{t_1}s''P_{t_1}\emptyset, \qquad s'P_{t_2}sP_{t_2}s''P_{t_2}\emptyset,$$
$$sP_{t_3}s'P_{t_3}s''P_{t_3}\emptyset, \qquad s''P_{t_4}sP_{t_4}s'P_{t_4}\emptyset.$$

*If in the first step of SI-CC school $s$ points to $t_1$, the best school $t_3$ can point is $s'$. Therefore, she will be assigned to $s'$. In particular, under true preferences, SI-CC assigns all employees to their status-quo schools. This outcome is not SI-constrained efficient because it is Pareto dominated by another status-quo improving matching $\nu$ for teachers where $\nu_{t_1} = \nu_{t_3} = s$, $\nu_{t_2} = s'$ and $\nu_{t_4} = s''$. Moreover, if $t_3$ swaps the rankings of $s'$ and $s''$, then SI-CC selects $\nu$, i.e., $t_3$ manipulates SI-CC when $s$ points $t_1$ in the first step.*

In Example A.3, we show that, in the same setting as Combe et al. (2022), SI-CC is not equivalent to the teacher optimal selection of TO-BE they propose.[87]

**Example A.3.** *Let $S = \{s_1, s_2\}$, $T = \{t_1, t_2, t_2'\}$, $\omega_{s_1} = \{t_1\}$, $\omega_{s_2} = \{t_2, t_2'\}$, $q_{s_1} = 1$ and $q_{s_2} = 2$. Let $\tau(t_2) \rhd_{s_1} \tau(t_2') \rhd_{s_1} \tau(t_1) \rhd_{s_1} \theta_\emptyset$ and $\tau(t_1) \rhd_{s_2} \tau(t_2) \rhd_{s_2} \tau(t_2') \rhd_{s_2} \theta_\emptyset$. Each teacher prefers the other school to her status-quo school.*

*One can check that the matching selected by the teacher optimal selection of TO-BE matches*[88] *$t_1$ to $s_2$ and $t_2$ to $s_1$ while SI-CC matches $t_1$ to $s_2$ but $t_2'$ to $s_1$.*

**Example A.4.** *Below we illustrate our concepts introduced in Section 4 as well as the arguments behind Proposition 1 and Corollary 1 using the T20/B20 ratio inequality index and mean statistic. We also illustrate the need for the large market assumption in these results.*

*Let $S = \{s_1, s_2\}$, $T = \{h_1, \ell_1, h_2, \ell_2\}$. The status-quo matching $\omega$ is given as*

$$\omega_{s_1} = \{h_1, \ell_1\}, \;\; \omega_{s_2} = \{h_2, \ell_2\},$$

[87] Combe et al. (2022) already noted that their class of TO-BE mechanisms do not entirely define the class of status-quo improving, strategy-proof and two-sided Pareto efficient mechanisms. However, they did not investigate it further. Our example suggests that other non-trivial mechanisms, such as SI-CC, exist outside their class.

[88] One can easily check that this example is well defined in their setting. Just set the preferences of the schools over the teachers being equivalent to the schools' ranking over their corresponding types.

*Let $q_{s_1} = q_{s_2} = 2$. The preferences of teachers are*[89]*:*

$$s_2 P_{h_1} s_1 P_{h_1} \emptyset$$
$$s_1 P_{\ell_1} s_2 P_{\ell_1} \emptyset$$
$$s_2 P_{h_2} s_1 P_{h_2} \emptyset$$
$$s_1 P_{\ell_2} s_2 P_{\ell_2} \emptyset.$$

*Let $\Theta = \{\theta_1, \theta_2, \theta_3\} = \{\ell, m, h\}$, and the letter name of each teacher is her type. We consider the following values for the types: (since there are no vacant seats we do not explicitly refer to the vacant seat type $\theta_\emptyset$)*

$$v_1 = v_\ell = 2, \qquad v_2 = v_m = 4, \qquad v_3 = v_h = 6.$$

*We let the profile of non-participating employees be given for any $n \geq 1$ as*

$$E^n_{s_1} = \{m^1_1, \ldots, m^n_1\},$$
$$E^n_{s_2} = \{\ell^1_2, \ldots, \ell^n_2\}.$$

*Suppose we use the mean statistic as $f$ and the T20/B20 index as the inequality index $\mathcal{I}$.*[90] *We start our illustration by constructing the natural type ranking profile of $\mathcal{I}$ and $f$. The non-participant profile value distribution $\delta^*$ such that for any school $s$ and type $k = 1, 2, 3$, $\delta^{*v_k}_s = \frac{|E^{\theta_k}_s|}{|E_s|}$, is as follows:*

$$\delta^*_{s_1} = (0,\ 1,\ 0)\,, \delta^*_{s_2} = (1,\ 0\ ,0)\,.$$

*The non-participant mean type value vector is found as follows:*

$$z^{*f} = f(\delta^*) = (4,\ 2)\,,$$

*leading to the following ordering of the statistics:*

$$z^{*f}_{s_2} < z^{*f}_{s_1}.$$

*The value of T20/B20 index at $z^{*f}$ is $4/2 = 2$. We have that the partial derivatives of $\mathcal{I}$ are well defined at $z^{*f}$ and one can easily verify that they are also ordered as*

$$\frac{\partial \mathcal{I}}{\partial z_{s_2}}(z^{*f}) < 0 < \frac{\partial \mathcal{I}}{\partial z_{s_1}}(z^{*f}).$$

[89]It may be seen as questionable for teachers to participate in the (re)assignment process while ranking all other schools as worse than their current one — such as teacher $\ell_1$ and $h_2$. One could easily adjust the example to ensure that these teachers rank other schools higher than their initial schools, but they fail to be assigned to any of those preferred schools.

[90]See Section 4 for the definition.

*Thus, we obtain the following partition of schools induced by $\mathcal{I}$ and $f$ at the base economy:*

$$L^{\mathcal{I},f} = \{s_2\} \qquad \text{and} \qquad H^{\mathcal{I},f} = \{s_1\},$$

*and the natural type ranking profile of $\mathcal{G}$ and $f$, denoted as $\trianglerighteq$, is*

$$\begin{gathered} h \rhd_{s_2} m \rhd_{s_2} \ell \\ \ell \rhd_{s_1} m \rhd_{s_1} h. \end{gathered}$$

*The outcome matching of SI-CC is $\mu$ such that*

$$\mu_{s_1} = \{\ell_1, \ell_2\}, \;\; \mu_{s_2} = \{h_1, h_2\}.$$

*Note that this is the unique (non-trivial) individually rational and status-quo improving matching for schools under the natural type ranking profile of $\mathcal{I}$ and $f$. Next, we show how much the outcome of SI-CC reduces the T20/B20 index with respect to the status quo for different n values (the replications of the base economy): First, we illustrate the status-quo value distribution for each school s, $\delta^{\omega}_s = (\delta^{\omega,2}_s, \delta^{\omega,4}_s, \delta^{\omega,6}_s)$:*

$$\delta^{\omega,n}_{s_1} = \left(\frac{1}{n+2}, \frac{n}{n+2}, \frac{1}{n+2}\right), \delta^{\omega,n}_{s_2} = \left(\frac{n+1}{n+2}, 0, \frac{1}{n+2}\right).$$

*This leads to a mean statistic vector of*

$$f(\delta^{\omega,n}) = \left(4, \frac{2n+8}{n+2}\right).$$

*We obtain the following value for the T20/B20 index at the status-quo matching $\omega$ in an n-economy:*

$$\mathcal{I}(f(\delta^{\omega,n})) = 4\frac{n+2}{2n+8}.$$

*Then we consider the outcome $\mu$ of SI-CC which leads to a value distribution profile*

$$\delta^{\mu,n}_{s_1} = \left(\frac{2}{n+2}, \frac{n}{n+2}, 0\right), \delta^{\mu,n}_{s_2} = \left(\frac{n}{n+2}, 0, \frac{2}{n+2}\right),$$

*and a mean statistic vector*

$$f(\delta^{\mu,n}) = \left(4\frac{n+1}{n+2}, 2\frac{n+6}{n+2}\right).$$

*We note that $4\frac{n+1}{n+2} \geq 2\frac{n+6}{n+2}$ if and only if $n \geq 4$. Hence, we obtain the following value for the T20/B20 index at the matching achieved by SI-CC in an n-economy:*

$$\mathcal{I}(f(\delta^{\mu,n})) = \left\{ \begin{array}{l} (n+6)/2(n+1) \text{ for } n \leq 4 \\ 2(n+1)/(n+6) \text{ for } n \geq 4 \end{array} \right\}.$$

*It is easy to check that $\mathcal{I}(f(\delta^{\mu,n})) < \mathcal{I}(f(\delta^{\omega,n}))$ if and only if $n > 2$. Next, we find the outcome of the T20/B20 index at $\omega$ and $\mu$ for different $n$-economies.*

| $n$ | *non-participant economy size* | *T20/B20 of the status quo* $\mathcal{I}(f(\delta^{\omega,n}))$ | *T20/B20 of SI-CC matching* $\mathcal{I}(f(\delta^{\mu,n}))$ | *T20/B20 variation* |
|---|---|---|---|---|
| 1 | 33.33% | 1.2 | 1.75 | +46% |
| 2 | 50% | 1.33 | 1.33 | +0% |
| 3 | 60% | 1.43 | 1.12 | −22% |
| 4 | 66.67% | 1.5 | 1 | −33% |
| 5 | 71.43% | 1.56 | 1.09 | −30% |
| 6 | 75% | 1.6 | 1.17 | −27% |
| 10 | 83.33% | 1.71 | 1.37 | −20% |
| 20 | 90.91% | 1.83 | 1.62 | −11% |
| 50 | 96.15% | 1.93 | 1.82 | −6% |
| $\infty$ | 100% | $\mathcal{I}(z^{*f}) = 2$ | | − |

*SI-CC achieves a more equitable outcome than the status-quo matching when the non-participants economy size is large enough (i.e., if it represents more than 50% of the whole market). When $n$ is smaller than 2, the school in $L^{\mathcal{I},f}$ surpasses the school in $H^{\mathcal{I},f}$ in terms of average experience of teachers under the SI-CC outcome and this shift is large enough that the inequality index increases. This shift in school status only occurs when the non-participants' economy size is relatively small and as the market size increases, SI-CC reduces inequality.*

# E Extension: Impossibilities under Alternative Status-quo Improvement Definition

In this section, we weaken restrictions on school ranking over teachers. Instead of the Lorenz domination relation for unambiguous weak improvement, we use a responsive order induced by schools ranking over the types of teachers (Roth, 1985). To this end, we first construct an order over teachers and vacancy option, denoted by $\emptyset$, for a given $\trianglerighteq_s$ and denote it by $\succsim_s^{\trianglerighteq_s}$. Let $\succ_s^{\trianglerighteq_s}$ and $\sim_s^{\trianglerighteq_s}$ denote the asymmetric and symmetric parts of $\succsim_s^{\trianglerighteq_s}$.

Given $\trianglerighteq_s$, the order of school $s$ over $T \cup \{\emptyset\}$ is given as:

- $\tau(t) \trianglerighteq_s \tau(t')$ if and only if $t \succsim_s^{\trianglerighteq_s} t'$;
- $\tau(t) \trianglerighteq_s \theta_\emptyset$ if and only if $t \succsim_s^{\trianglerighteq_s} \emptyset$.

For any $|\bar{T}| < q_s$ responsiveness implies that for any $t, t' \in T \setminus \bar{T}$

- $\bar{T} \cup \{t\} \succsim_s^{\trianglerighteq_s} \bar{T}$ if and only if $t \succsim_s^{\trianglerighteq_s} \emptyset$;
- $\bar{T} \cup \{t\} \succsim_s^{\trianglerighteq_s} \bar{T} \cup \{t'\}$ if and only if $t \succsim_s^{\trianglerighteq_s} t'$.

Note that, responsive order is more general than Lorenz preferences. In particular, if $\mu_s$ Lorenz dominates matching $\omega_s$, then $\mu_s \succsim_s^{\rhd_s} \omega_s$. However, the other way may not be true. We illustrate this in the following example.

**Example A.5.** *Let $T = \{t_1, t_2, t_3, t_4, t'_1, t'_4\}$, $S = \{s\}$, $\omega_s = \{t_1, t_2, t_3, t_4\}$, and $\tau(t_1) = \tau(t'_1) \rhd_s \tau(t_2) = \tau(t_3) \rhd_s \tau(t_4) = \tau(t'_4)$. Consider the following matching $\mu_s = \{t_1, t'_1, t_4, t'_4\}$. Matching $\mu_s$ does not Lorenz dominate $\omega_s$. However, it is possible that $\mu_s \succ_s^{\rhd_s} \omega_s$.*

The following example shows that, when unambiguous (and therefore, status-quo) improvement is defined based on responsive orders, there is no mechanism that is constrained efficient and strategy-proof.

**Example A.6.** *There are six teachers, $T = \{t_1, t'_1, t_2, t'_2, t, t'\}$, and four schools, $S = \{s_1, s_2, s, s'\}$. Schools do not have vacant seats. Let $\omega_{s_1} = \{t_1, t'_1\}$, $\omega_{s_2} = \{t_2, t'_2\}$, $\omega_s = \{t\}$ and $\omega_{s'} = \{t'\}$. Let $\unrhd$ be the type ranking profile such that:*

$$\tau(t) \rhd_{s_1} \tau(t_1) \rhd_{s_1} \tau(t'_1) \rhd_{s_1} \tau(t') \rhd_{s_1} \theta_\emptyset \rhd_{s_1} \tau(t_2) \rhd_{s_1} \tau(t'_2)$$
$$\tau(t') \rhd_{s_2} \tau(t_2) \rhd_{s_2} \tau(t'_2) \rhd_{s_2} \tau(t) \rhd_{s_2} \theta_\emptyset \rhd_{s_2} \tau(t_1) \rhd_{s_2} \tau(t'_1)$$
$$\tau(t_1) \rhd_s \tau(t_2) \rhd_s \tau(t'_2) \rhd_s \tau(t'_1) \rhd_s \tau(t') \rhd_s \tau(t) \rhd_s \theta_\emptyset$$
$$\tau(t_1) \rhd_{s'} \tau(t_2) \rhd_{s'} \tau(t'_2) \rhd_{s'} \tau(t'_1) \rhd_s \tau(t) \rhd_{s'} \tau(t') \rhd_{s'} \theta_\emptyset.$$

*For notational simplicity, we use $\succsim_s$ instead of $\succsim_s^{\rhd_s}$ for all $s \in S$, Moreover, we assume that $\{t, t'\} \succ_{s_k} \{t_k, t'_k\}$ for $k \in \{1, 2\}$. Notice that, this relation is consistent with responsiveness. Preferences of the teachers are:*

$$s_2 \, P_t \, s_1 \, P_t \, s \, P_t \, \emptyset, \qquad s_1 \, P_{t'} \, s_2 \, P_{t'} \, s' \, P_{t'} \, \emptyset,$$
$$s \, P_{t_1} \, s_1 \, P_{t_1} \, \emptyset, \qquad s' \, P_{t'_1} \, s_1 \, P_{t'_1} \, \emptyset,$$
$$s \, P_{t_2} \, s_2 \, P_{t_1} \, \emptyset, \qquad s' \, P_{t'_2} \, s_2 \, P_{t'_2} \, \emptyset.$$

*First note that under any status-quo improving matching, if $t$ is assigned to her first ranked school $s_2$, then $t'$ must also be assigned to $s_2$. Indeed, let $\mu$ be a status-quo improving matching such that $\mu_t = s_2$. Since $\{t, t'\} \succ_{s_2} \{t_2, t'_2\} \succ_{s_2} \{t_2, t\} \succ_{s_2} \{t'_2, t\}$, status-quo improvement implies that $\mu_{t'} = s_2$. With a similar argument, if $\mu_{t'} = s_1$, then $\mu_t = s_1$. So it implies that there are only* *three possible constrained efficient matchings:*

$$\mu^1 := \begin{pmatrix} t & t' & t_1 & t_1' & t_2 & t_2' \\ s_1 & s_1 & s & s' & s_2 & s_2 \end{pmatrix}$$
$$\mu^2 := \begin{pmatrix} t & t' & t_1 & t_1' & t_2 & t_2' \\ s_2 & s_2 & s_1 & s_1 & s & s' \end{pmatrix}$$
$$\mu^3 := \begin{pmatrix} t & t' & t_1 & t_1' & t_2 & t_2' \\ s_1 & s_2 & s & s_1 & s_2 & s' \end{pmatrix}$$

*Let $\varphi$ be a constrained efficient mechanism. Assume that $\varphi(P) = \mu^1$. In that case, let $P_t'$ : $s_2 \, P_t' \, s \, P_t' \, \emptyset \, P_t' \, s_1$. Under $(P_t', P_{-t})$, the only constrained efficient matching is $\mu^2$ so that $\varphi(P_t', P_{-t}) = \mu^2$ and the manipulation of $t$ is successful. If $\varphi(P) = \mu^2$, then $t'$ can report $P_{t'}'$ : $s_1 \, P_{t'}' \, s' \, P_{t'}' \, \emptyset \, P_{t'}' \, s_2$ so that the only constrained efficient matching under $(P_{t'}, P_{-t'})$ is $\mu^1$ and $\varphi(P_{t'}, P_{-t'}) = \mu^1$, a successful manipulation for $t'$. If $\varphi(P) = \mu^3$ then $t$ or $t'$ can manipulate in reporting the same profile as before. We conclude that $\varphi$ cannot be strategy-proof.*

# F Descriptions and Theoretical Results Pertaining to Other Mechanisms used in the Empirical Analysis

## F.1 TTC*

Technically, TTC* is an extension of the YRMH-IGYT mechanism of Abdulkadiroğlu and Sönmez (1999) to the case when there are multiple positions at a school. Most notably, a school's two positions can be assigned simultaneously in two different ways when a chain is executed in a step: A vacant position can be assigned through the chain in which it is the last school, and an occupied position can be filled by an incoming teacher while its status-quo employee for that position is assigned to a different school.

**Definition A.1.** ***The TTC* Mechanism*** *Let $\vdash$ be a tiebreaker over tenured teachers and $\vdash^*$ be a tiebreaker over all teachers. For each school $s$, we construct a* ***pointing order*** *$\gtrdot_s$ over teachers in $\omega_s$ using its type rankings $\trianglerighteq_s$ and $\vdash$: For any two distinct teachers $t, t' \in \omega_s$*

$$t \gtrdot_s t' \iff \tau(t) \lhd_s \tau(t') \text{ or } [\tau(t) \sim_s \tau(t') \text{ and } t \vdash t'].$$

*The second tiebreaker $\vdash^*$ will be used for chain selection below. It is a separate tiebreaker as in TTC*, chains can be started by any teacher, i.e., not only by new teachers, and thus, choice of it presents another policy tool.*

*A general Step $k$ is defined as follows:*

***Step** $k$**:***

- *Each remaining school $s$ points to the highest priority remaining teacher in $\omega_s$ under $\gtrdot_s$, if not* *all students in $\omega_s$ are already removed. Otherwise, school s does not point to any teacher.*

- *Each remaining teacher t points to her most preferred remaining option.*
- *Outside option $\emptyset$ points to all teachers pointing to it.*

*Due to finiteness, there exists either*

*(i) a cycle, or*

*(ii) a chain.*

*Then:*

- ***If Case (i) holds:** Each teacher can be in at most one cycle as she points at most to a single option. We execute exchanges in each cycle by assigning the teachers in that cycle to the school she points to, remove assigned teachers and filled schools, and go to Step $k+1$.*
- ***If Case (i) does not hold:** Then case (ii) holds, i.e., there exists a chain. In particular, each remaining teacher initiates a chain. Then we select the chain such that the first teacher of the chain is the highest priority teacher under **chain tiebreaker** $\vdash^*$ and the last school of the chain is one that does not point to a teacher.*

  *We execute the exchanges in the selected chain by assigning each teacher in the chain to the school she points to, remove assigned teachers and filled schools, and go to Step $k+1$.*

*The mechanism terminates when all teachers are removed.*

## F.2 The Current French Mechanism

The Current French Mechanism uses a version of teacher proposing Gale and Shapley (1962) deferred-acceptance (DA) algorithm using school priority orders mandated by the French Ministry of Education. In the next subsection, we explain how these priorities are determined. The mechanism modifies these mandated priorities so that it is an individually rational mechanism for tenured teachers. Other than this modification, the priority relations of schools remain unchanged among the status-quo teachers and the non-status-quo teachers. Then the mechanism utilizes the DA algorithm using these modified school priorities and the submitted teacher preferences. Also see Guillen and Kesten (2012), Pereyra (2013), and Compte and Jehiel (2008) regarding the use of this algorithm in another context and teacher assignment context. This mechanism is strategy-proof for teachers. See Combe et al. (2022) for a more detailed presentation of this mechanism and its properties.

### F.2.1 Determinants of the Current Ministry-Mandated Priorities in France

This subsection of the appendix presents the criteria used by the French Ministry of Education in 2013 to prioritize teachers.

**Priorities that apply to all teachers**

- **Seniority in the profession**. The number of points teachers receive depends on their "class". Teachers move up class during their career:
  - Normal Class (early career): Teachers receive 7 points per grade level, with a minimum of 21 points in total, i.e., a flat rate of 21 points for the first, second, and third grades. See Figure A.12 for a correspondence between the number of years of experience and grade level.
  - Upper Class (mid-career): Teachers get 49 flat-rate points and 7 points per grade level in the upper class. Aggregated teachers in the upper class at the sixth-grade level can claim 98 points if they have two years of seniority at that level.
  - Exceptional Class (end-of-career): Teachers get 77 flat-rate points and 7 points per grade level in the exceptional class, up to a maximum of 98 points.
- **Seniority in the current school**. 10 points are awarded per year of service in the current position as a tenured employee. An additional 25 points are awarded for each four-year period of seniority in the position.

**Figure A.12:** Correspondence Between Experience and Grade Level

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Normal class** | **Grade** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** |
| (early career) | Duration (in years) | 1 year | 1 year | 2 years | 2 years | 2.5 years | 3 years | 3 years | 2.5 years | 4 years | 4 years |
| **Upper class** | **Grade** | **1** | **2** | **3** | **4** | **5** | **6** | | | | |
| (mid career) | Duration (in years) | 2 years | 2 years | 2.5 years | 2.5 years | 3 years | 3 years | | | | |
| **Exceptional class** | **Grade** | **1** | **2** | **3** | **4** | **5** | **6** | | | | |
| (end of career) | Duration (in years) | 2 years | 2 years | 2.5 years | 3 years | 1 year | 1 year | | | | |

Notes: This figure shows, for each grade level, the number of years a teacher is expected to stay in the grade level before moving to the following one.

### Priorities defined by law

- **Teachers requesting spousal reunification**: 150.2 points are awarded for the spouse's region and neighboring regions. 100 points are granted for each dependent child under the age of 20. Additional points are granted by years of separation: 190 points are granted for the first year of separation, 325 points for two years of separation, 475 points for three years, and 600 points for four years or more.
- **Disability**: 1,000 points are awarded for the regions where the requested transfer will improve the situation of the disabled person.
- **Priority education (Affectation à caractère prioritaire justifiant une valorisation (A.P.V.))**: Teachers who spent several years in a school labeled as A.P.V receive 300 points for five to seven years or 400 points for eight years or more.

### Priorities based on personal or administrative situation

- **Teachers in a replacement zone**: Teachers in replacement zones (T.Z.R.) who requested and obtained a permanent position in a school receive a bonus of 100 points if they stay five years in the assigned school. This bonus cannot be combined with a bonus granted under the A.P.V. (Priority Education Area) scheme.
- **Trainees**: A bonus of 0.1 point is awarded to candidates for their first assignment when they request the academy where they completed their first-year training.
- **Teachers re-entering teaching after a break**: A bonus of 1,000 points is awarded for the region where they were employed before their break.
- **Simultaneous transfer between two teachers**: A flat-rate bonus of 80 points is awarded for the region entered as the first choice and the neighboring regions.
- **Proximity to the child's residence**: In cases of shared custody, and to support housing and visitation rights, a bonus of 150 points is awarded, applicable to the first choice and neighboring regions. The first request must correspond to the region where the child resides.
- **High-level athletes temporarily assigned to the region where they have their sporting interests**: A bonus of 50 points is awarded for each successive year of temporary assignment, for up to four years, and applies to all region requests made.
- **Teachers assigned to French Guiana**: Teachers assigned to French Guiana will, after five years in this oversees region, receive a bonus of 100 points on each of their requested regions.

**Priorities based on the expressed preference**

- **Preferential request**: A bonus of 20 points per each consecutive year is granted starting from the year when the teacher expresses the same first regional preference as the one expressed the previous year.[91]
- **Assignment in overseas departments (DOM) or Mayotte**: A bonus of 1,000 points is awarded for first-choice requests concerning the academies of Guadeloupe, French Guiana, Martinique, Réunion, or the vice-rectorate of Mayotte, for teachers who can justify the presence of their center of material and moral interests (CIMM) in that department.
- **Single preference for the region of Corsica**: Bonuses are awarded for the preference "Corsica," provided that the candidate has made this single request. The bonus for the single "Corsica" preference is progressive: 600 points for the first request, 800 points for the second consecutive request, and 1,000 points for the third consecutive request

[91]Priorities based on teacher expressed preference make the current French mechanism non-strategy-proof. Our analysis never assumes that the current mechanism is strategy-proof. In particular, when estimating teacher preferences, to avoid the potential bias generated by teachers misreporting their preferences, we estimate the preferences of teachers under a weaker *stability assumption* developed by Fack et al. (2019) and applied to the teacher assignment by Combe et al. (2022).

and beyond.

### F.3 SI-CC*

Let $S^{CVA}$ denote the subset of regions including Créteil, Versailles, and Amiens, the three youngest regions. Given a market $P$, type ranking profile $\trianglerighteq$, and a tiebreaker $\vdash$, SI-CC* selects its outcome through the following steps.

**Step 1: Pseudo Preference Profile Construction**

We construct a pseudo preference profile $P^*$ from $P$ as follows:

For each $t \notin N$, $P_t^* = P_t$. For each $t \in N$:

- $s \; P_t^* \; s'$ for all $s \in S^{CVA}$ and $s' \notin S^{CVA}$, and
- $s \; P_t^* \; s'$ if and only if $s \; P_t \; s'$ for all $s, s' \in S^{CVA}$ (or $s, s' \notin S^{CVA}$).

**Step 2: Running SI-CC under Pseudo Preference Profile**

The matching selected by SI-CC under $P^*$, $\trianglerighteq$, and $\vdash$ is the outcome of SI-CC*.

Now, we are ready to analyze the properties of SI-CC*.

**Proposition A.5.** *For any type ranking profile $\trianglerighteq$, SI-CC* is strategy-proof and status-quo improving for schools under $\trianglerighteq$. Moreover, it is individually rational if all new teachers rank regions in $S^{CVA}$ over $\emptyset$.*

**Proof.** Since the outcome of SI-CC* is found by running the SI-CC algorithm for any given $\trianglerighteq$ and the preference profile of the teachers does not affect whether the selected outcome is status-quo improving for schools or not, Theorem 1 implies that SI-CC* is status-quo improving for schools. Moreover, by Theorem 1, no teacher is assigned to a school ranked below $\emptyset$ in pseudo preferences under the outcome of SI-CC*. Hence, by our construction of pseudo preference under SI-CC*, if all new teachers rank regions in $S^{CVA}$ over $\emptyset$, then the outcome of SI-CC* is individually rational.

Next, we show SI-CC* is strategy-proof. Fix a preference profile $P$. First of all, if $P_t^* = P_t$, then Theorem 1 implies that teacher $t$ cannot be better off by misreporting her preference order under SI-CC*. Now consider a teacher $t$ such that $P_t^* \neq P_t$. By our construction, $t \in N$. Recall that, the outcome of SI-CC* is found by running SI-CC under $P^*$. By the definition of SI-CC, any new teacher is assigned through a chain, there exists exactly one new teacher in any executed chain, and a chain is executed when there does not exist a cycle. Hence, independent of the preference orders submitted by the new teacher, SI-CC will execute the same set of cycles until no cycle exists to execute. Let $k$ be the first step of SI-CC under $P^*$ in which there is no cycle to execute. Without loss of generality, let $t$ be the new teacher with the highest tiebreaker under $\vdash$. By construction, the regions in $S^{VCA}$ are ranked at the top of $P_t^*$. By definition, $t$ will be assigned to the region, possibly $\emptyset$, she points to in Step $k$. Suppose $t$ points to $s$. By the construction of the pseudo preference profile and the definition of SI-CC, $t$ cannot point to a more preferred school than $s$ in Step $k$ of SI-CC* by misreporting her preferences. Hence, $t$ cannot benefit from misreporting.

Since teachers assigned in the latter steps cannot change the assignments done in the first $k$ steps, we can follow the same reasoning to show that no teacher can manipulate SI-CC*.

Despite the properties satisfied by SI-CC*, it is not constrained efficient, as illustrated in the following example.

**Example A.7.** *Let $S = \{s, s'\}$, $s \in S^{CVA}$, $s' \notin S^{CVA}$, $N = T = \{t_1, t_2\}$, $q_s = q_{s'} = 1$, and $\tau(t_1) = \tau(t_2) \trianglerighteq_{\bar{s}} \theta_\emptyset$ for all $\bar{s} \in S$. The preferences of the teachers are*

$$s \; P_{t_1} \; s' \; P_{t_1} \; \emptyset$$
$$s' \; P_{t_2} \; s \; P_{t_2} \; \emptyset$$

*Suppose $t_2 \vdash t_1$. Then, SI-CC* selects matching $\mu$ such that $\mu(t_1) = s'$ and $\mu(t_2) = s$. Matching $\mu$ is Pareto dominated by another status-quo improving matching $\nu$ such that $\nu(t_1) = s$ and $\nu(t_2) = s'$.*

# G Empirical Analysis: Estimation of Teacher Preferences

**The Estimation Model.** We estimate the preferences of teachers over regions using the following utility function:

$$u_{t,r} = \delta_r + Z'_{t,r}\beta + \varepsilon_{t,r} \tag{24}$$

Teacher $t$'s utility for region $r$ is a function of region fixed effect $\delta_r$, teacher-region-specific observables $Z_{t,r}$ (with coefficient vector $\beta$) and a random shock $\varepsilon_{t,r}$ which is i.i.d. over $t$ and $r$ and follows a type-I extreme value distribution, Gumbel$(0, 1)$. The region fixed effect captures region characteristics such as the average socio-economic and academic level of students, cultural activities, housing prices, facilities, etc. We estimate preferences separately for tenured teachers and new teachers. This allows us to include a richer set of variables for the former group. The vector $Z_{t,r}$ includes a dummy specifying whether region $r$ is the birth region of teacher $t$. If teacher $t$ is tenured, it also includes a dummy showing whether $r$ is the status-quo region of teacher $t$ and the distance between region $r$ and the status-quo region of teacher $t$. Vector $Z_{t,r}$ additionally includes interaction terms between teacher $t$'s and region $r$ school characteristics (that are presented in Panels A and B of Table A.1). We apply standard scale and position normalization, setting the scale parameter of the Gumbel distribution to 1 and the fixed effect of the Paris region to 0.

**Identifying assumptions.** To avoid the potential bias generated by teachers omitting regions they consider as infeasible, we estimate the preferences of teachers under a weaker *stability assumption* developed by Fack et al. (2019) and applied to the teacher assignment by Combe et al. (2022).[92]

We start by defining the *feasible set* of each teacher as the set of regions with a cutoff—that is, the lowest priority of the teacher assigned to a region—lower than her own priority. These are the regions a teacher could be assigned to if she ranked the region first in her preference list. The key identifying assumption is that, for each teacher, the region obtained is her most preferred region among all regions that are in her feasible set.[93] Hence, we estimate a discrete choice model with personalized choice sets. Choice probabilities have closed form solutions, and we estimate parameters using maximum likelihood.

**Estimation results.** Table A.5 reports preference estimates for tenured and new teachers for a selected group of coefficients. We run the estimations in each of the eight subjects separately and report results for Math and French teachers in the table. The first nine rows report coefficients for a selected set of region fixed effects. They reveal an interesting difference between the preferences of tenured and new teachers. While the Créteil and Versailles regions are very unattractive for tenured teachers (as indicated by the negative coefficient of their fixed effect relative to the Paris region), in each of the eight subjects we consider (except Sports), these regions are more attractive for new teachers.[94] The fact that Créteil and Versailles are more attractive for new teachers, who often see a first position in a priority education school as a stepping stone for better positions in the future, than for tenured teachers surely contributes to the unequal distribution of teachers denounced by policymakers.[95] Yet, this is not the only explanation for teacher unequal distribution. The counterfactual analysis we present in Section 5 shows that the assignment mechanism also shapes the distribution of teachers in important ways. The fact that preferences alone do not drive the unequal distribution is fundamental for our ability to improve both teacher distribution and teacher welfare. Appendix J reports goodness of fit measures for preference estimation.

**Simulations.** We use our estimates of utility coefficients to draw preferences of teachers 1,000 times using Equation (24). After having drawn them, we keep the entire set of regions without imposing any truncation for their simulated preference lists so that teach-

[92]Combe et al. (2022) provide an in-depth discussion of the two alternative identifying assumptions (truthfulness versus stability), as well as statistical tests in favor of the latter. They mainly focus on estimating the preferences of tenured teachers, and we use the same estimation in our analysis. In this paper, we provide an additional detailed discussion on the estimation of the preferences of new teachers. For more references on estimations that do not require truth-telling, see Akyol and Krishna (2017), Artemov et al. (2019), Agarwal and Somaini (2018), and Calsamiglia et al. (2020).

[93]This assumption is theoretically founded: Artemov et al. (2019) show that, in a large market environment, any (regular) equilibrium outcome of a mechanism class that includes the Current French mechanism must have this property.

[94]The 16 coefficients for the regions of Créteil and Versailles are always more negative for tenured teachers than for new teachers (except in Sports). Still, the difference is not always statistically significant because of sample sizes.

[95]As mentioned before, teachers who stay in a priority education school for at least five years benefit from additional priority when they ask to change region or school.

**Table A.5:** Teacher Preference Estimates

| | Tenured Teachers | | | | New Teachers | | | |
|---|---|---|---|---|---|---|---|---|
| | French | | Math | | French | | Math | |
| | coef. (1) | s.e. (2) | coef. (3) | s.e. (4) | coef. (5) | s.e. (6) | coef. (7) | s.e. (8) |
| Region BESANCON | -5.95*** | (1.09) | 0.11 | (0.69) | -2.49** | (0.81) | 0.04 | (1.12) |
| Region BORDEAUX | -2.76** | (1.06) | 0.74 | (0.58) | 1.60* | (0.72) | 0.58 | (0.95) |
| Region DIJON | -6.44*** | (1.08) | -3.57*** | (0.68) | -2.70*** | (0.73) | -2.97** | (0.93) |
| Region LILLE | -7.09*** | (1.07) | -1.52 | (0.79) | -3.60*** | (0.75) | -1.59 | (1.00) |
| Region REIMS | -7.80*** | (1.12) | -3.77*** | (0.66) | -4.54*** | (0.74) | -4.23*** | (0.96) |
| Region AMIENS | -7.99*** | (1.17) | -3.06*** | (0.66) | -4.31*** | (0.73) | -3.57*** | (0.92) |
| Region ROUEN | -7.38*** | (1.08) | -3.08*** | (0.62) | -3.94*** | (0.73) | -1.92* | (0.94) |
| Region CRÉTEIL | -7.38*** | (1.10) | -3.79*** | (0.67) | -3.15*** | (0.74) | -3.04*** | (0.88) |
| Region VERSAILLES | -6.14*** | (1.01) | -2.69*** | (0.57) | -3.00*** | (0.69) | -2.70** | (0.86) |
| Status-quo region | 1.88 | (6.56) | -26.25** | (8.00) | | | | |
| Birth region | 8.50** | (3.19) | 11.42*** | (3.32) | 8.62** | (2.75) | 8.49** | (3.01) |
| Dist. to status-quo region | -17.98*** | (4.30) | -23.03*** | (4.75) | | | | |
| % stud. urban x Status-quo region | -5.28*** | (0.83) | -4.72*** | (1.06) | | | | |
| % stud. urban x Teach. from CV | 2.53*** | (0.69) | 0.90 | (0.62) | | | | |
| % stud. in priority ed. x Married | -5.26*** | (1.51) | -1.71 | (1.58) | -0.55 | (1.85) | 3.42* | (1.64) |
| % stud. in priority ed. x Status-quo region | 9.74*** | (2.78) | 3.52 | (3.58) | | | | |
| % stud. in private sch. x Teach. in disadv. sch. | -1.14 | (1.84) | 2.84 | (1.76) | | | | |
| % teach. younger than 30 x Advanced qualif. | 15.61*** | (3.54) | -3.19 | (3.04) | 11.11* | (4.37) | -0.33 | (2.66) |
| % teach. younger than 30 x Status-quo region | 42.49*** | (4.90) | 37.63*** | (5.76) | | | | |
| % teach. younger than 30 x Birth region | -19.69*** | (3.53) | -18.80*** | (4.77) | -8.59** | (3.02) | -6.14* | (2.81) |
| Region in South of France x Teach. from CV | -1.43*** | (0.37) | 0.30 | (0.34) | | | | |
| Number of teachers | 859 | | 605 | | 786 | | 958 | |
| Fit measure | 0.654 | | 0.601 | | 0.625 | | 0.604 | |

Notes: This table reports selected coefficients from estimations of teacher preference for region characteristics based on Equation 24. We set the fixed effect of the Paris region to 0. The last row reports our goodness of fit measure that we compute by looking at the top two regions that a teacher has included in her submitted preference list. We measure, for each teacher, the probability of observing this particular preference ordering in the preference list predicted with our estimations. We then average these probabilities across teachers. Stars correspond to the following p-values: * $p< 0.05$; ** $p< 0.01$; *** $p< 0.001$. Variable "Teach. from CV" refers to whether the status-quo region of the teacher is Créteil or Versailles.

ers find all regions acceptable, while for the tenured teachers individual rationality of the mechanisms implies being assigned to a region no worse than her status-quo region.[96] In

[96]This implicit assumption about new teachers is in line with the policy of the Ministry. Teachers are

each of the eight subjects and for each draw, we use these simulated preferences to run the mechanisms. The results reported in Section 5 correspond to averages over these 1,000 draws, aggregated over eight subjects.

# H Empirical Analysis: Alternative Inequality Indexes and Statistics

The results presented so far use the T20/B20 ratio inequality index and the mean teacher experience statistic in each region to identify high-type and low-type regions. We show in this section that we reach similar conclusions on inequality reductions under SI-CC when using three alternative combinations of indices and statistics to measure inequality:

1. The T20/B20 ratio index based on the share of teachers that have more than four years of experience (rather than the mean experience). Policymakers care about the share of inexperienced teachers in each region because it can lead to achievement inequalities among students (Bates et al., 2021, Chetty et al., 2014, and Rockoff, 2004), but also because assigning many new teachers to undesirable regions contributes to the lack of attractiveness of the profession (Cour des Comptes, 2013, 2017).
2. The maximin index (also known as the Atkinson-$\infty$ index) based on teacher mean experience. The Atkinson-$\varepsilon$ index Atkinson (1970) is based on a social welfare approach and is a function of the society's inequality aversion parameter $\varepsilon \geq 0$. Given the statistic vector $z = (z_s)_{s \in S}$, it is defined as the normalized ratio of the equally distributed equivalent level of statistics to the $(w_s)_{s \in S}$-weighted mean of the actual statistic distribution across regions as $\mathcal{A}^{\varepsilon}(z) = 1 - \frac{\left(\sum_{s \in S} w_s z_s^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}}{\sum_{s \in S} w_s z_s}$. The index lies between zero and one and increases with rising inequality. Varying the aversion to social inequality parameter $\varepsilon$ can achieve a different index sensitivity to the lower tail of the distribution of $z$. As $\varepsilon$ increases, the social planner cares more about the inequality regarding the lower tail. The varying aversion $\varepsilon$ can be important in our teacher-assignment context depending on the policymaker's objectives. The limit $\varepsilon \to \infty$ refers to the Rawlsian inequality aversion, which aims at increasing the lowest region's statistic as much as possible (hence, the name maximin index follows for this particular case). In our robustness check, we use this index, which aligns with the aforementioned objectives of the French Ministry of Education of decreasing the experience gap and is calculated when $\varepsilon \to \infty$ as $A^{\infty}(z) = 1 - \frac{\min_{s \in S} z_s}{\sum_{s \in S} w_s z_s}$.
3. Maximin index based on the share of teachers that have more than four years of experience.

indeed not required to rank all regions when they submit their lists, but the Ministry fills the incomplete lists of new teachers to make sure that all of them get an assignment; even those who ranked few regions.

**Table A.6:** Alternative Inequality Indexes and Statistics

| | Status quo | Main mech. | | Other mech. | |
|---|---|---|---|---|---|
| | | SI-CC | TTC* | SI-CC* | Current French |
| | (1) | (2) | (3) | (4) | (5) |
| T20/B20(Mean experience) | 1.3588 | 1.3487 | 1.3691 | 1.3489 | 1.3829 |
| *% change compared to status-quo* | | [-2.79%] | [+2.87%] | [-2.75%] | [+6.72%] |
| T20/B20(Share $>$ 4 years exp) | 1.227 | 1.219 | 1.239 | 1.217 | 1.260 |
| *% change compared to status-quo* | | [-3.75%] | [+5.35%] | [-4.18%] | [+14.46%] |
| Maximin(Mean experience) | 0.2345 | 0.2221 | 0.2376 | 0.2282 | 0.2487 |
| *% change compared to status-quo* | | [-5.28%] | [+1.32%] | [-2.69%] | [+6.05%] |
| Maximin(Share $>$ 4 years exp) | 0.1640 | 0.1532 | 0.1698 | 0.1586 | 0.1869 |
| *% change compared to status-quo* | | [-6.54%] | [+3.58%] | [-3.29%] | [+14.03%] |

For each of these three alternative measures of inequalities, we replicate the entire process of (1) defining high-type and low-type regions based on the sign of the derivative of the inequality index, (2) customizing the regions' teacher type rankings based on whether they are in $L$ (i.e., low-type) or in $H$ (i.e., high-type), and finally (3) running counterfactuals using teacher preferences and type ranking profiles. We then compute the inequality index for each mechanism.

We report the results in Table A.6.[97] Two facts stand out. First, SI-CC reduces the inequality for all index-statistic combinations compared to the status quo. In contrast, the Current French mechanism substantially worsens inequalities for all index-statistic combinations. Second, the reduction of inequality (in percentage) between SI-CC and the status quo is larger when considering the maximin index than when considering the T20/B20 ratio index. This interesting difference is because set $L$ consists of only the lowest-statistic region while all other regions are classified as $H$ under the maximin index, and hence by the opposing nature of the type rankings between these two sets, the lowest-statistic region gets a high inflow of experienced teachers. Under the T20/B20 ratio index, usually at least two-three regions are in $L$ (even if we classify all zero-derivative and positive-derivative regions as $H$).

[97] As we did in the main text, for the T20/B20 index, we describe the change in the experience difference ratio. For example, for the statistic share of teachers with more than four years of experience, the status-quo experience difference ratio is 22.7%, and the Current French mechanism increases it to 26%. Then the change in this ratio is about +14.5%.

# I Empirical Analysis: Robustness Checks for $L$ and $H$ Region Classifications under T20/B20 Ratio Index

This Appendix reports robustness checks in which we vary the definition of low-type and high-type regions. As explained in Section 5.3, for our empirical analysis, in each subject, regions are partitioned into two groups: $H$ and $L$. The regions in $H$ (or $L$) are referred to as *high-type* (or *low-type*). For this partition, we compute the T20/B20 ratio and its partial derivative for each region at the status quo based on the experience of all teachers. We then use the sign of the index partial derivative to create two groups: positive-partial derivative regions are assigned to $H$, and negative-partial derivative regions are assigned to $L$. The T20/B20 ratio index also has regions with zero-partial derivative. In the counterfactual results presented in the paper, those regions are assigned to $L$ if their mean experience is below the median region's and otherwise assigned to $H$.

In Table A.7, we simulated what happens if we assign the regions with zero-partial derivative to $L$ (column 2) or to $H$ (column 3), respectively. These two alternative specifications provide very similar results in terms of overall mobility (4,910 under the baseline classification used throughout the paper versus 4,888 under the *low-type* classification and 4,734 under the *high-type* classification). The *high-type* classification performs better than our baseline specification in terms of overall inequality reductions. The T20/B20 ratio of teacher experience is equal to 1.3487 with our baseline specification. It goes down to 1.3406 under the *high-type* classification. This good performance comes from the particularly low number of new teachers with no experience who are assigned to the B20 regions (155 under the high-type classification versus 665 under our baseline classification).

# J Empirical Analysis: Variables Used for Teacher Preference Estimations and Goodness of Fit Measures

**Variables used for teacher preference estimation.** The way they are abbreviated in Table A.5 is written in parentheses.

We use the following region characteristics:

- Share of students classified as disadvantaged.
- Share of students living in an urban area as % (labeled as "% stud. urban").
- Share of students who attend a school classified as priority education (labeled as "% stud. in priority ed."). Priority education is a label given to the most disadvantaged schools in France.
- Share of students who attend a private school (labeled as "% stud. in private sch.").
- Share of teachers who are younger than 30 (labeled as "% teach. younger than 30")
- Region is in South of France (labeled as "Region in South of France"). The follow-

**Table A.7:** SICC - Alternative Classifications for High-Type and Low-Type Regions

| | Zero index derivative classified as: | | |
|---|---|---|---|
| | Baseline | Low-type | High-type |
| | (1) | (2) | (3) |
| | Panel A. Inequality Index | | |
| Ratio T20/B20 *Value at status quo = 1.3588* | 1.3487 | 1.3505 | 1.3406 |
| | Panel B. Teacher mobility | | |
| Tenured teachers moved and new teachers assigned | 4,910 | 4,888 | 4,734 |
| Tenured teachers moved from the T20 regions | 152 | 270 | 280 |
| Tenured teachers moved from the B20 regions | 172 | 148 | 180 |
| Tenured teachers moved from all regions | 986 | 964 | 810 |
| New teachers assigned | 3,924 | 3,924 | 3,924 |
| New teachers unassigned | 713 | 713 | 713 |
| New teachers (0 exp) assigned to the B20 regions | 665 | 777 | 155 |
| New teachers (exp $>$ 0) assigned to the B20 regions | 445 | 321 | 887 |

ing 5 regions are classified as being in the South of France: Aix-Marseille, Bordeaux, Montpellier, Toulouse, and Nice.

We use the following teacher characteristics:

- Current region of the teacher (labeled as "Status-quo region"). This is the region a teacher is initially assigned to.
- Region where a teacher was born (labeled as "Birth region").
- Distance between the region ranked and the status-quo region of a teacher (labeled as "Distance to status-quo region").
- Teacher's current region is Créteil or Versailles, which are the two least attractive regions (labeled as "Teach. from CV"). The attractiveness of a region is measured by the ratio of the number of teachers who rank the region as their first choice divided by the number of teachers who ask to leave the region.
- Teacher is married (labeled as "Married").
- Teacher has spent at least 5 years in a school labelled as priority education (labeled as "Teach. in priority education").
- Teacher has an advanced teaching qualification (labeled as "Advanced qualif.").

**Goodness of fit measures.** Our main fit measure (also reported in Table A.5) considers the top two regions that a teacher has included in her submitted preference list. We then compute the probability of observing this particular relative ordering in the preference list predicted by our estimations. This fit measure based on relative ranking (instead of the characteristics of the school ranked first, for instance) is particularly suitable for our environment in which some teachers might not rank regions that they consider as infeasible.[98] In addition to the overall fit quality, we also compute fit measures for the tenured teachers who are employed in the two least attractive regions, namely, Créteil and Versailles, at the status quo. Inspecting the fit quality for this sub-group of teachers is particularly important because teachers from Créteil and Versailles represent a large share of the tenured teachers who submit a transfer request every year and they are more likely to stay in their positions. These two facts could affect the preference estimation for these teachers under our fairness assumption. Across the 8 subjects, our fit measures range from 0.62 to 0.72 for tenured teachers and from 0.56 to 0.69 for new teachers, which compare favorably to those obtained by Fack et al. (2019) (between 0.553 and 0.615).

[98]When teachers skip regions perceived as infeasible, the first region they report might not be their most preferred region—and indeed, the tests we perform reject truth-telling—but conditional on ranking schools, the order in which a teacher ranks the schools might correspond to teacher true relative preference. This is why we prefer to use a fit measure that is based on relative ranking rather than on the characteristics of the school ranked first.