Priorities vs. Precedence in School Choice: Theory and Evidence from Boston*

Umut Dur Scott Duke Kominers Parag A. Pathak Tayfun Sönmez†

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Abstract

A central issue in school choice is the debate over who obtains seats at oversubscribed schools. Choice plans in many cities grant students higher priority for some seats at their neighborhood schools. This paper demonstrates how the precedence order, i.e. the order in which seats are depleted by applicants with specific claims, is a lever to achieve distributional goals that has effects comparable to priorities under the deferred acceptance algorithm. While Boston gives priority to neighborhood applicants in half of the seats at each school, the intended effect of this policy is almost fully lost because of the precedence order of the seats; its outcome is nearly equivalent to that of a mechanism without any neighborhood priority. This fact shows how the precedence order can undermine the intended role of priorities. A change in precedence, holding fixed the current 50-50 school seat split, corresponds to almost three-quarters of the effect of switching from 0% to 100% neighborhood priority. We formally establish that either increasing the number of neighborhood priority seats at a school or adjusting the precedence of neighborhood seats have the same qualitative effect: an increase in the number of neighborhood students assigned to that school. Decisions about precedence have distributional effects with little impact on the overall number of students who receive their top choices and therefore are inseparable from decisions about priorities.

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†Dur: Department of Economics, University of Texas at Austin, email: umutdur@gmail.com. Kominers: Becker Friedman Institute, University of Chicago, email: skominers@uchicago.edu. Pathak: Massachusetts Institute of Technology and NBER, Department of Economics, email: ppathak@mit.edu, Sönmez: Department of Economics, Boston College, email: sonmezt@bc.edu.

1 Introduction

School choice reforms aspire to sever the link between the housing market and access to good schools. In residence-based public school systems, families can purchase access by living in communities with desirable schools, leaving children from families less able or willing to live in these neighborhoods without this opportunity. By allowing families the option to choose outside their neighborhoods, choice plans offer the potential to more widely distribute access to sought-after schools. Opening up competition, however, requires policies that specify how seats for students from both inside and outside of neighborhoods will be rationed. Many U.S. cities have provisions so that both groups have a shot at attending in-demand schools.

Whether seen through the prism of riots over court-ordered busing, contentious discussions on zone definitions within cities, or squabbles over the size of explicit reservations at particular schools, the tension between those who want to attend their neighborhood schools and those who want to leave their neighborhoods is at the heart of many school choice debates. For example, Boston's current system of student assignment emerged out of a federal judge's 1974 decision to mandate racial balance across schools by busing students across neighborhoods. This system has recently been the center of an intense city-wide discussion following the 2012 State of the City address by Boston Mayor Thomas Menino.[1] In the speech, Menino (2012) articulated support for the faction in favor of greater neighborhood assignment:

> "Something stands in the way of taking our [public school] system to the next level: a student assignment process that ships our kids to schools across our city. Pick any street. A dozen children probably attend a dozen different schools. Parents might not know each other; children might not play together. They can't carpool, or study for the same tests. [...] Boston will have a radically different school assignment process – one that puts priority on children attending schools closer to their homes."

Critics expressed concerns that families from disadvantaged neighborhoods would be shut out of good schools if the neighborhood component of assignment is given more weight. This counterpoint is summarized by a community activist (Seelye 2012):

> "A plan that limits choice and that is strictly neighborhood-based gets us to a system that is more segregated than it is now."

Understanding the implications of policies striving for a compromise between these two factions presents challenges without an adequate assignment mechanism. A first ingredient for determining access to good schools is the set of applicants' claims or property rights for school seats. The controversial issue of who among the applicants is deserving not only involves traditional economic considerations, but also hotly-contested moral and ethical concerns inherent in problems of distributive justice. As a result, these aspects of school assignment have mostly been taken

[1] For more on this debate, see the materials available at http://bostonschoolchoice.org and press accounts by Goldstein (2012) and Handy (2012).

as given in the literature on school choice mechanism design and have not been the focus of design efforts in the field.[2] A second ingredient is how students' claims are processed through the assignment mechanism. The interpretation of student property rights and issues related to their processing is perhaps one reason why the theoretical literature on student assignment (Balinski and Sönmez 1999, Abdulkadiroğlu and Sönmez 2003) took nearly forty years to develop following pioneering contributions on two-sided matching (Gale and Shapley 1962).

Choice plans based on variants of the student-proposing deferred acceptance algorithm allow for the design of property rights to be considered in isolation from the mechanics of determining the allocation. Other mechanisms often lack this complete separation between property rights and participant choices. For instance, in the old (pre-2005) Boston mechanism, if two students have equal priority for a school, their preference ranking of the school determines whose claim is more justified. In the deferred acceptance algorithm, in contrast, the notion of a property right does not depend on student choices. As a result, by setting school **priorities**, such as giving higher claims to sibling or neighborhood applicants, districts can precisely define property rights for applicants independent of demand under mechanisms based on deferred acceptance.

Specifying priorities is only one part of determining access, however. Another aspect involves specific set-asides for particular types of applicants. Currently, Boston Public Schools (BPS) splits schools equally into two pieces: at one half students from the neighborhood obtain priority, while at the other half neighborhood priority does not play a role. Boston's 50-50 school seat split emerged after the city adopted a race-neutral plan for assignment in 1999 and has been in place for the last 12 years (Daley 1999a).[3] When students rank a school, they are considered for both types of slots.[4] The processing order of these slots, or their **precedence**, determines how seats are depleted by applicants. Daley (1999b) reports that the 50-50 plan was initially seen as "striking an uneasy compromise between neighborhood school advocates and those who want choice," while the Superintendent had hoped that the plan "will satisfy both factions, those who want to send children to schools close by and those who want choice."

We begin our investigation by using data on choices and assignments from Boston Public Schools to examine whether the current mechanism indeed represents a compromise. We compare the current BPS outcome to two alternatives: one where none of the seats have the neighborhood, i.e. "walk-zone" priority, and one where all seats have walk-zone priority. Given the 50-50 split and its motivation, it is natural to expect that the outcome will be close to the middle of these two opposite policies. However, Table 1 shows that the current BPS mechanism is surprisingly close to the outcome where walk-zone priority is not used at all. Only 3% of Grade K1 applicants obtain a different assignment under Boston's current implementation than they would under open competi-

[2]For example, a December 2003 community engagement process in Boston considered six different proposals for alternative neighborhood zone definitions. However, the only recommendation adopted by the school committee was to switch the assignment algorithm (Abdulkadiroğlu, Pathak, Roth, and Sönmez 2005).

[3]The 50-50 school seat split was not altered when Boston changed their assignment mechanism in 2005 to one based on the student-proposing deferred acceptance algorithm (Abdulkadiroğlu, Pathak, Roth, and Sönmez 2005, Abdulkadiroğlu, Pathak, Roth, and Sönmez 2006, Pathak and Sönmez 2008).

[4]Throughout this paper, we use slot and seat interchangeably.

tion with no walk-zone priorities, as indicated by the column labeled 0% Walk. Furthermore, this pattern is not simply a feature of student demand. Compared to the alternative where all seats have walk-zone priority, labeled 100% Walk, the number of students assigned to a school in their walk zone increases by 19% and 17% for the two main elementary school entry points (Grades K1 and K2). Although motivated as a compromise between the two factions, BPS's 50-50 school seat split is much closer to open competition than the impression a 50-50 split might at first suggest.

This paper is about understanding this puzzle: Why does BPS's mechanism result in an assignment so close to one without any neighborhood priority, even though half of school seats give priority to neighborhood students? To answer this question, we develop a framework for school choice mechanism design where both priority and precedence play key roles. The 50-50 division of slots between a priority structure with and without neighborhood priorities reveals little about the proximity of the outcome to completely open competition and 100% Walk without specifying how the allocation process proceeds when a student is qualified for both types of slots. Building on Kominers and Sönmez (2012), we establish two new comparative static results: (1) given a fixed slot precedence order, replacing an open slot with a neighborhood slot at a school weakly increases the number of neighborhood students assigned to that school, and (2) given a fixed split of seats into neighborhood and open slots, switching the precedence order position of a neighborhood slot with the position of an open slot weakly increases the number of neighborhood students assigned to that school.

While these features are intuitive, the rich slot-specific priority structure of our economy implies that they do not follow from immediate generalizations of the respect for improvements property in simpler models without the slot structure (Balinski and Sönmez 1999). These results show that the precedence order has a significant influence on the eventual assignment and in extreme cases can virtually eliminate the intended effect of having variations in slots' priorities. We further specialize to a two-school model where we show that above mentioned priority and precedence order changes not only weakly increase the number of neighborhood assignments at the school having the change, but also weakly increase the neighborhood assignments across all schools. This impact is entirely distributional as both instruments leave the aggregate number of students who obtain a top choice unchanged.

Next, we empirically examine the extent to which the comparative statics from the simpler model provide practical insight for the richer set of priorities in Boston's school choice plan. After demonstrating that Boston's current implementation of the 50-50 split is far from the midpoint between neighborhood and choice proponents, we show that an alternative precedence order where non-neighborhood slots are depleted before neighborhood slots increases the number of students who attend a walk-zone school by 8% in Grade K1. This represents more than two-thirds of the maximal achievable difference between completely eliminating walk-zone priority and having walk-zone priority apply at all school seats. As a result, in Boston, precedence order details have quantitative impacts comparable to the entire range of possible adjustments to the number of walk-zone slots. Finally, we examine alternative precedence orders which implement intermediate positions between the two extremes of depleting all neighborhood seats first and depleting open seats first. Adopting one of these intermediate precedence orders would restore the "compromise" role of the 50-50 school seat split. This is, perhaps, the most important policy implication of our paper.

This paper contributes to a broader agenda, examined in a number of recent papers, that introduces concerns for diversity into the literature on school choice mechanism design (see, e.g., Budish, Che, Kojima, and Milgrom (2011), Echenique and Yenmez (2012), Erdil and Kumano (2012), Hafalir, Yenmez, and Yildirim (2012), Kojima (2012), and Kominers and Sönmez (2012)). When an applicant ranks a school with many seats, it is similar to expressing indifference among the school's seats. Therefore, our work parallels recent papers examining the implications of indifferences in school choice problems (Erdil and Ergin 2008, Abdulkadiroğlu, Pathak, and Roth 2009, Pathak and Sethuraman 2011). However, the question of school-side indifferences, the focus of prior work, is entirely distinct from the issue of indifferences in student preferences. Tools used to resolve indifferences for schools (like random lotteries) do not immediately apply for the student side. Another somewhat related paper is Roth (1985), who considers interpreting a college admissions model (many-to-one), through an expansion of a marriage model (one-to-one), where the many side is split into pieces and applicants rank pieces in a given order. We show using data from Boston Public Schools that improper implementation of this interpretation can result in unintentionally undermining the intention of particular priority policies. Finally, this paper builds on the theoretical literature on matching with contracts (Crawford and Knoer 1981, Kelso and Crawford 1982, Hatfield and Milgrom 2005, Ostrovsky 2008, Hatfield and Kojima 2010, Echenique 2012) and the applied motivation shares much with recent work on matching in the military (Sönmez and Switzer 2011, Sönmez 2011).

The paper proceeds as follows. Section 2 introduces the model and illustrates the roles of precedence and priority. Section 3 reports on an empirical investigation of these issues in the context of Boston's school choice plan. The last section concludes. All proofs are relegated to the appendix.

2 Model

There is a finite set of students I and a finite set of schools A. Each school a has a finite set of slots S^a. We use the notation a_0 to denote a "null school" to represent the possibility of being unmatched; we assume that this option is always available to all students. Let $S \equiv \bigcup_{a \in A} S^a$ denote the set of all slots (excluding those at the null school). We assume that $|S| \geq |I|$, so that there are enough (real) slots for all students. Each student i has a strict preference relation P^i over A. Throughout the paper we fix the set of students I, the set of schools A, the set of schools' slots S, and the students' preferences $(P^i)_{i \in I}$.

For any school $a \in A$, each slot $s \in S^a$ has a linear priority order π^s over students in I. This linear priority order captures the "property rights" of the students for this slot in the sense that the higher a student is ranked under π^s, the stronger claims he has for the slot s of school a. Following the current practice in BPS, we allow slot priorities to be heterogeneous across slots of a given school. A subtle consequence of this within-school heterogeneity is that we must determine how slots are assigned when a student is "qualified" for multiple slots with different priorities at a school. The last primitive of the model regulates this selection in a linear way: For each school $a \in A$, the slots in S^a are ordered according to a (linear) **order of precedence** $\triangleright^a$. Given a school $a \in A$ and two of its slots $s, s' \in S^a$, the expression $s \triangleright^a s'$ means that slot s is to be filled before slot s' at school a whenever possible.

A **matching** $\mu : I \to A$ is a function which assigns a school to each student such that no schools is assigned to more students than its total number of slots. Let μ_i denote the assignment of student i, and μ_a denote the set of students assigned to school a.

Our model generalizes the school choice model of Abdulkadiroğlu and Sönmez (2003) in that it allows for heterogenous priorities across the slots of a given school. Nevertheless, a mechanism based on the celebrated *student-proposing deferred acceptance algorithm* easily extends to this model once the *choice function* of each school is constructed for given slot priorities and order of precedence.

Given a school $a \in A$ with a set of slots S^a, a list of slot priorities $(\pi^s)_{s \in S^a}$, an order of precedence $\triangleright^a$ with

$$s_a^1 \triangleright^a s_a^2 \triangleright^a \cdots \triangleright^a s_a^{|S^a|},$$

and a set of students $J \subseteq I$, the **choice of school a from the set of students J** is denoted by $C^a(J)$, and is obtained as follows: Slots at school a are filled one at a time following the order of precedence $\triangleright^a$. The highest priority student in J under $\pi^{s_a^1}$, say student j_1, is chosen for slot s_a^1 of school a; the highest priority student in $J \setminus \{j_1\}$ under $\pi^{s_a^2}$ is chosen for slot s_a^2 of school a, and so on.

For a given list of slot priorities $(\pi^s)_{s \in S}$ and an order of precedence $\triangleright^a$ at each school $a \in A$, the outcome of the **student-proposing deferred acceptance mechanism (DA)** can be obtained as follows:

***Step 1*:** Each student i applies to her top choice under P^i. Each school a with a set of Step 1 applicants J_1^a tentatively holds the applicants in $C^a(J_1^a)$, and rejects the rest.

In general at Step ℓ,

***Step ℓ* :** Each student who is rejected at Step $\ell - 1$ applies to her next choice school. Each school a considers its new applicants together with those on hold from Step $\ell - 1$, and uses its choice function C^a to determine which students are tentatively held and which students are rejected.

The algorithm terminates when no additional student is rejected.

2.1 A Mix of Neighborhood-Based and Open Priority Structures

In this paper we are particularly interested in the slot priority structure used at Boston Public Schools. There is a master priority order π^o that is uniform across all schools. This master priority order is obtained via an even lottery and is often referred to as the **random-tiebreaker**. At each school in Boston, slot priorities depend on students' walk-zone and sibling statuses and the random-tiebreaker π^o. For our theoretical analysis, we will consider a simplified version which only depends on walk-zone status and the random-tiebreaker. We show in our empirical analysis of Section 3 that this is a good approximation for Boston Public Schools.

For any school $a \in A$, there is a subset $I_a \subset I$ of **walk-zone students** that is determined with a concrete formula. There are two types of slots:

1. **Walk-zone** slots: For each walk-zone slot at a school a, any walk-zone student $i \in I_a$ has priority over any non-walk-zone student $j \in I \setminus I_a$, and the priority order within these two groups is determined with the random tie-breaker π^o.

2. **Open** slots: $\pi^s = \pi^o$ for each open slot s.

For any school $a \in A$, define S_w^a to be the set of walk-zone slots and S_o^a to be the set of open slots. BPS currently uses a DA where half of the slots at each school are walk-zone slots, while the remaining half are open slots. This structure has been historically interpreted as a compromise between the proponents of neighborhood assignment and the proponents of school choice.

An important comparative statics exercise concerns the impact of replacing an open slot with a walk-zone slot under DA for a given order of precedence. One might naturally expect such a change to weakly increase the number of students who are assigned to a walk-zone school. Surprisingly, this is not correct in general as we show in the next example.

Example 1. There are four schools $A = \{k, l, m, n\}$. Each school has two available slots. There are eight students $I = \{i_1, i_2, i_3, i_4, i_5, i_6, i_7, i_8\}$. Let I_a be the students living in the walk-zone of school $a \in A$. There are two walk-zone students at each school. Let $I_k = \{i_1, i_2\}$, $I_l = \{i_3, i_4\}$, $I_m = \{i_5, i_6\}$ and $I_n = \{i_7, i_8\}$. The random tie-breaker π^o orders the students as:

$$\pi^o : i_1 \succ \; i_8 \succ i_3 \succ i_4 \succ i_5 \succ i_6 \succ i_7 \succ i_2.$$

The preference profile is:

P^{i_1}	P^{i_2}	P^{i_3}	P^{i_4}	P^{i_5}	P^{i_6}	P^{i_7}	P^{i_8}
k	k	l	l	m	m	n	k
l	l	k	k	k	k	k	l
m	m	m	m	l	l	l	m
n	n	n	n	n	n	m	n

.

First consider the case where each school has one walk-zone slot and one open slot. Also assume that the walk-zone slot has higher precedence than the open slot at each school.

The outcome of DA for this case is:

$$\mu = \begin{pmatrix} i_1 & i_2 & i_3 & i_4 & i_5 & i_6 & i_7 & i_8 \\ k & n & l & l & m & m & n & k \end{pmatrix}.$$

Observe that six students (i.e. students $i_1, i_3, i_4, i_5, i_6, i_7$) are assigned to their walk-zone schools in this scenario.

Next we replace the open slot at school k with a walk-zone slot, so that both slots at school k are walk-zone slots. Each remaining school has one slot with walk-zone priority and one slot with open priority, with the walk-zone slot having higher precedence than the open slot.

The outcome of DA for the second case is:

$$\mu' = \begin{pmatrix} i_1 & i_2 & i_3 & i_4 & i_5 & i_6 & i_7 & i_8 \\ k & k & l & m & m & n & n & l \end{pmatrix}.$$

Observe that five students (i.e. students i_1, i_2, i_3, i_5, i_7) are assigned to their walk-zone schools in the second case. That is, the total number of walk-zone assignments decreases when the open slot at school k is replaced with a walk-zone slot. □

Nevertheless, as we present next, replacement of an open slot with a walk-zone slot at a given school a weakly increases the number of walk-zone students assigned to school a (even though it may decrease the total number of walk-zone assignments).

Proposition 1. *For any given order of precedence of slots, replacing an open slot with a walk-zone slot at school a weakly increases the number of walk-zone students who are assigned slots of school a under DA.*

When a school district increases the fraction of walk-zone slots, one of the policy motives behind this change is to increase the fraction of students assigned to walk-zone schools. As we have shown in Proposition 1, replacing an open slot with a walk-zone slot serves this goal through its "first-order effect" in the school directly affected by the change, although the overall effect across all schools might in theory be in the opposite direction. Nevertheless, our empirical analysis in the next section using data from BPS suggests that the first-order effect dominates – the overall effect is in the expected direction.

While the role of the number of walk-zone slots as a policy tool is quite clear, the role of the order of precedence is much more subtle. Indeed, the choice of the order of precedence is often considered a minor technical detail, and until now it has never entered policy discussions. In this paper, we show not only that the choice of the order of precedence has important distributional implications, but that its effect is very substantial in the case of BPS.

Qualitatively the effect of decreasing the order of precedence of a walk-zone slot is similar to the effect of replacing an open slot with a walk-zone slot. While this may appear counter-intuitive at first, the reason is simple: By decreasing the order of precedence of a walk-zone slot, one increases the odds that a walk-zone student who has high enough priority for both types of slots is assigned to an open slot rather than a walk-zone slot. This in turn increases the competition for the open slots and decreases the competition for the walk-zone slots. Our next result formalizes this observation.

Proposition 2. *Fix the set of walk-zone slots and the set of open slots at each school. Then, switching the order of precedence position of a walk-zone slot at school a with that of a subsequent open slot at school a weakly increases the number of walk-zone students who are assigned to school a under DA.*

Given Example 1, it is not surprising to see that the aggregate effect of such a change across all schools may contradict its "first order" effect. Example 2 we present next is a modified version of Example 1 making this point.

Example 2. To illustrate the conceptual relation between priority swaps and changes in the order of precedence, Example 2 closely follows Example 1. The only difference is a small modification in the second case. There are four schools $A = \{k, l, m, n\}$. Each school has two available slots. There are eight students $I = \{i_1, i_2, i_3, i_4, i_5, i_6, i_7, i_8\}$. Let I_a be the students living in the walk zone of school $a \in A$. There are two walk-zone students at each school. Let $I_k = \{i_1, i_2\}$, $I_l = \{i_3, i_4\}$, $I_m = \{i_5, i_6\}$ and $I_n = \{i_7, i_8\}$. The random tie-breaker π^o orders the students as:

$$\pi^o : i_1 \succ i_8 \succ i_3 \succ i_4 \succ i_5 \succ i_6 \succ i_7 \succ i_2.$$

The preference profile is:

P^{i_1}	P^{i_2}	P^{i_3}	P^{i_4}	P^{i_5}	P^{i_6}	P^{i_7}	P^{i_8}
k	k	l	l	m	m	n	k
l	l	k	k	k	k	k	l
m	m	m	m	l	l	l	m
n	n	n	n	n	n	m	n

First consider the case where each school has one walk-zone slot and one open slot. Also assume that the walk-zone slot has higher precedence than the open slot at each school.

The outcome of DA for this case is:

$$\mu = \begin{pmatrix} i_1 & i_2 & i_3 & i_4 & i_5 & i_6 & i_7 & i_8 \\ k & n & l & l & m & m & n & k \end{pmatrix}.$$

Observe that six students (i.e. students $i_1, i_3, i_4, i_5, i_6, i_7$) are assigned to their walk-zone schools in this scenario.

Next change the order of precedence at school k so that its open slot has higher precedence than its walk-zone slot. Each remaining school maintains the original order of precedence with the walk-zone slot higher precedence than the open slot.

The outcome of DA for the second case is:

$$\mu' = \begin{pmatrix} i_1 & i_2 & i_3 & i_4 & i_5 & i_6 & i_7 & i_8 \\ k & k & l & m & m & n & n & l \end{pmatrix}.$$

Observe that five students (i.e. students i_1, i_2, i_3, i_5, i_7) are assigned to their walk-zone schools in the second case. That is, the total number of walk-zone assignments decreases as the precedence of the walk-zone slot at school k is reduced. □

2.2 Additional Results for the Case of Two Schools

In this section, we obtain sharper theoretical results by focusing on the case of two schools ($|A| = 2$). We assume that each student belongs to one walk zone and that students rank both schools. This case is motivated in part by the commonly discussed policy objective of giving students from poorer neighborhoods access to desirable schools in richer neighborhoods.

We have the following additional results for this case.

Proposition 3. *Suppose there are two schools. For either school and any order of precedence of its slots, replacing an open slot with a walk-zone slot weakly increases the total number of walk-zone assignments under DA.*

An immediate implication of Proposition 3 is the following intuitive result justifying the ideal policies of the two polar factions.

Corollary 1. *Suppose there are two schools and the number of slots is fixed at both schools. Under DA:*

- *The minimum number of walk-zone assignments across all priority and precedence policies is obtained when all slots have open slot priority, and*
- *the maximum number of walk-zone assignments across all priority and precedence policies is obtained when all slots have walk-zone priority.*

Proposition 4. *Suppose there are two schools. Fix the set of walk-zone slots and the set of open slots at each school. Then, switching the order of precedence position of a walk-zone slot at school a with that of a subsequent open slot at school a weakly increases the total number of walk-zone assignments under DA.*

While the precedence alone does not cover the entire spectrum of outcomes reached via priority adjustment, it may cover a significant part as we present in our empirical analysis in Section 3. Moreover, the fraction of students who receive their first choices, second choices, and so forth show virtually no response to changes in the fraction of walk-zone slots or the order of precedence. Our next result provides a theoretical basis for this empirical observation.

Proposition 5. *Suppose there are two schools. The number of students assigned to their top choice schools is independent of both the number of walk-zone slots and the choice of precedence order.*

An important policy implication of our last result is that the division of slots between walk-zone priority and open priority as well the order of precedence selection has little bearing on the aggregate number of students who receive their top choices; thus, the impact of these DA calibrations on student welfare is mostly distributional.

3 Empirical Analysis

Examples 1 and 2 illustrate that determining how many students attend a walk-zone school for particular priority structures or precedence orders is challenging without additional structure on priorities or preferences. These examples motivate the two-priority-type model presented in the last section. When we further simplify our model to two schools, Propositions 3 and 4 show that changes to either precedence or priority weakly increase the total number of students who obtain a walk-zone assignment.

To examine whether these comparative static predictions capture the main features of school choice with richer priority structures, we use data on submitted preferences from Boston Public Schools. In contrast to our two-priority-type model, there are three additional priority groups in Boston: (1) guaranteed applicants, who are typically continuing on at their current school, (2) sibling-walk applicants, who have siblings currently attending a school and live in the walk zone, and (3) sibling applicants, who have siblings attending a school and live outside the walk zone. In Boston, guaranteed students are ordered ahead of sibling-walk applicants, who in turn are ordered ahead of sibling applicants for both school halves. A single random number is used to order students within a priority group.

We use data covering four years from 2009-2012 when BPS employed a mechanism based on the student-proposing deferred acceptance algorithm. Students interested in enrolling in or switching schools are asked to list schools each January for the first round. Students entering kindergarten can either apply for elementary school at Grade K1 or Grade K2 depending on whether they are four or five years old. Since the mechanism is based on the student-proposing deferred acceptance algorithm and there is no restriction on the number of schools that can be ranked, the assignment mechanism is strategy-proof.[5] BPS also informs families of this property on the application form where it advises families to "list your school choice in your true order of preference. If you list a popular school first, you won't hurt your chances of getting your second choice school if you don't get your first choice" (BPS 2012). Since the mechanism is strategy-proof, we can isolate the effects of changes in priorities and precedence holding submitted preferences fixed.

As a check on our understanding of the data, we verify that we can recreate the assignments produced by BPS. Table A1 reports the fraction of students who have the same exact assignment under BPS and our recreation of the BPS assignment. Across four years and three applicant grades, we can match 98% of the assignments, with little variation across grades and years.[6] For what follows, we therefore take our recreation of BPS as representing the BPS assignment.

The motivating puzzle for this paper is shown in Table 1, which reports a comparison of the assignment produced by BPS, which relies on a 50-50 split of slots, to two extreme alternatives

[5] For analysis of the effects of restricting the number of choices which can be submitted, see (Haeringer and Klijn 2009, Calsamiglia, Haeringer, and Kljin 2010, Pathak and Sönmez 2011).

[6] Based on discussions with BPS, we learned that the reason why we do not exactly recreate the BPS assignment is that we do not have access to BPS's exact capacity file, and instead must construct it ex-post based on the final assignment. There are small differences between this measure of capacity and the capacity input to the algorithm due to the handling of unassigned students who are administratively assigned.

representing the ideal positions of these factions: (1) a priority structure without walk-zone priorities at any slot and (2) a priority structure where walk-zone priority applies at all slots. We refer to these two policies as **0% Walk** and **100% Walk**, and we simulate their outcomes using the same random numbers as BPS. Table 1 shows that the BPS assignment is very close to the former of these two polar alternatives and it differs for only 3% of students. One might suspect that this phenomenon to be driven by strong preferences among students for neighborhood schools.[7] Such preferences would bring the outcomes of all assignment policies close to each other. However, comparing the BPS outcome to 100% Walk, nearly 20% of students in Grade K1 obtain a different assignment. Therefore, the remarkable proximity of the current BPS outcome to the ideal of school choice proponents is not merely a reflection of negligible stakes in the choice of these policies. The increase in the number of students obtaining a walk-zone assignment shows that the comparative static prediction for priority changes presented in Proposition 3 is relevant for the more general environment of BPS. At Grade K2 and Grade 6, the fraction of students who obtain a different assignment under the 100% Walk alternative is 17% and 10%, respectively. The differences are smaller at higher grades because more students are continuing and therefore obtain guaranteed priority. On average, 4.5% of K2 applicants have guaranteed priority at their first choice, while 13% of Grade 6 students do. Hence, despite the adoption of a seemingly neutral 50-50 split, Table 1 shows the proximity of the BPS outcome to the 0% Walk outcome is present across different years and grades.

Another way to measure the relationship between the BPS assignment and 0% Walk is from the school perspective. In Figure 1, we compare the BPS assignment to 0% Walk and 100% Walk, and plot the fraction of students who obtain the same assignment with either extreme. In this figure, for instance, a value of 1 means that each student at the school receives the same outcome as in the BPS assignment. At 30 out of 63 Grade K1 schools, the assignments are exactly the same between 0% Walk and BPS. In contrast, the assignments are the same at only 5 schools between 100% Walk and BPS. Figures 2 and 3 report the same school-by-school comparisons for Grade K2 and Grade 6, and reinforce the pattern in Figure 1.

In Table 2, we consider the effects of alternative priority and precedence policies by reporting the number of students who are assigned to a school where they obtain walk-zone priority. The difference in fraction from the walk zone between 0% Walk (column 2) and the BPS assignment (column 4) is small. For Grades K1, K2, and 6, and taking 0% Walk as the benchmark only 1.0%, 1.2% and 0.6% more students obtain a walk-zone assignment under the BPS assignment, respectively. Table 2 also shows that the BPS assignment produces an outcome very close to that produced under the precedence policy **Walk-NonWalk**, which has all applicants first apply to walk-zone slots before applying to open slots.[8] The difference between columns (3) and (4) averages 0.6% across the three grades.

[7] Overall, 55.5% percent of students in our dataset rank a school where they obtain sibling-walk or walk priority first.

[8] Actual BPS implementation is a minor variant of Walk-NonWalk precedence where applicants with sibling priority and outside the walk zone apply to the open slots before applying to the walk-zone slots.

The close relationship between the BPS assignment and that produced by Walk-NonWalk precedence order provides a route towards understanding why the role of walk-zone priority is so limited under the current BPS practice. Under the Walk-NonWalk precedence order, applicants deplete slots at the walk-zone half first. If there are more walk-zone applicants than slots, then applicants' random numbers determine who obtains slots. Walk-zone applicants who do not obtain slots at the walk-zone half, therefore, systematically have less favorable random numbers than non-walk-zone applicants. When these applicants are considered for slots at the open half, their adversely selected random numbers place them behind applicants without walk-zone priority. As a result, a walk-zone applicant is unlikely to obtain a slot at the open half. This bias in applicant random numbers – created by the precedence order – renders the outcomes under Walk-NonWalk precedence very similar to the assignment that arises when all slots are open. Since the BPS outcome is so close to a 50-50 split with Walk-NonWalk precedence, this logic underlies why the BPS outcome bears close resemblance to the outcome without any role for walk-zone priority.

Having established that the BPS assignment is close to the assignment without walk-zone priority, we now explore alternative implementations of the 50/50 slot split. Corollary 1 to Proposition 3 suggests that the fraction of students who obtain an assignment in the walk zone under the 0% Walk and 100% Walk policies provides a benchmark for what can be obtained under variations of priority or precedence policy given student demand. For Grade K1, this range spans from 46.2% to 57.4% walk-zone assignment; the 11.2% interval represents the maximum range attainable through changes in either priorities or precedence. Columns (3)-(8) report five different implementations which all maintain the 50/50 school slot split. The Walk-NonWalk precedence represents a precedence policy that is at one end of the spectrum. The **NonWalk-Walk** precedence, under which all applicants first apply to open slots before applying to slots in the walk-half, represents the other end of precedence policy spectrum. The difference between these two policies can be seen as representing the range of possibilities, all given the 50-50 split, that can arise from alternative precedence orders.

Changes in precedence order have an impact that is comparable in magnitude to the effect of changing priorities. For Grade K1, the fraction of students who are assigned to a walk-zone school for the Walk-NonWalk and NonWalk-Walk precedence policies is 46.5% and 54.8%, respectively. The 8.3% difference between these two extremes corresponds to three-quarters of the 11.2% potential difference between the two polar cases of 0% Walk and 100% Walk. For Grade K2, the two opposite precedence policies cover 74% of the 9.3% range involving the two walk-zone priority policy extremes, while for Grade 6, the two opposite precedence policies cover 67% of the 5.4% range covering the two walk-zone priority policy extremes. Therefore, decisions about precedence order have welfare implications much like decisions about priorities.

What are possible precedence order policies with less partisan welfare implications? We report on three possible variations. First, we consider a precedence order which alternates walk-zone and open slots. The outcome of this **Rotating** treatment is shown in column (5). The fraction of students who are assigned to a school in their walk zone increases by 2.5% relative to the Walk-NonWalk precedence policy for Grade K1, but it is still biased towards it relative to the NonWalk-

Walk precedence policy. The reason that Rotating is closer to the Walk-NonWalk precedence is that alternating slots only partly undoes the bias created by the processing the walk-zone slots first. The pool of walk-zone applicants with favorable random numbers will be depleted and after the first few slots are allocated at the school, the bias in the pool of applicants assigned to the open slots re-emerges.

Another moderate alternative is to process half of the walk-zone slots first, followed by the entire open half, and then the other half of the walk-zone slots. This precedence order, which we label **Compromise**, attempts to reduce the bias created by processing all of the walk-zone slots first. By processing only half of the walk-zone slots, the competition between walk-zone and non-walk-zone applicants for open slots is more even. Initially, when the first few open slots are processed, the applicant pool has adversely selected random numbers, but this bias becomes less important by the time the last few open slots are processed. As a result, the fraction of applicants who attend a school for which they obtain walk-zone priority is close the midpoint between Walk-NonWalk and NonWalk-Walk. The Compromise policy attempts to even out the treatment of walk-zone applicants through changes in the order of slots.

The last alternative, which we label **Balanced**, breaks the bias in random numbers created by precedence order by using two random numbers. The first random number is used for walk-zone slots, while the second random number is used for open slots. To handle the issue with the differing pools of applicants from inside and outside the walk zone, we return to the rotating variation where a walk-zone slot is processed first, then an open slot, followed by a walk-zone slot, and so on. As a result, the Balanced precedence policy is like Rotating, except there are two distinct random numbers, one for each type of slot. Table 2 shows that the Balanced precedence policy leads to a greater number of students obtaining a walk-zone school. It is closer to the NonWalk-Walk outcome than the Walk-NonWalk outcome. For instance, for Grade K1, 51.7% of students are walkers, which is 3.1% less than the NonWalk-Walk outcome, but 5.2% greater than the Walk-NonWalk outcome. On the other hand, for Grade K1, the Balanced policy is close to the mid-point between 0% Walk and 100% Walk. For Grades K2 and 6, the Balanced policy is also closer to the mid-point between the two extreme variations on priorities than the mid-point between the two extreme precedence orders.

To understand how outcomes under the alternatives we consider are influenced by demand patterns, in Figure 4, we report the fraction of students assigned using walk-zone priority for underdemanded and overdemanded schools. An underdemanded school is one where there are fewer first choice applicants than slots, while an overdemanded school is one where there are more first choice applicants than slots. For Grade K1, roughly half of school programs are overdemanded and underdemanded with this definition. For Grade K2 and Grade 6, roughly 40% of schools are overdemanded. In Figures 4, 5, and 6, the fraction of students who are assigned to their walk-zone schools increases as move from left to right, with the total range greater at overdemanded schools than underdemanded schools.

Finally, the last issue we examine empirically is whether a version of Proposition 5 approximately holds for data from 2010-2011. We turn to examining how the overall distribution of choices received varies with precedence order in Table 3. This table shows that there is almost no difference in the aggregate distribution of choice rank received across the variations in precedence. Therefore, consistent with Proposition 5, changes in precedence are a tool to achieve distributional objectives, having little overall impact on the total number of students who obtain top choices.

4 Conclusion

The tension between those who want to attend schools in their neighborhoods and those who want to leave their neighborhoods is a key aspect of school choice policy debates in Boston and elsewhere. One perspective is in favor of having students attend schools close to their homes. Another perspective wants to ensure that families have high-quality choices, even if those choices are outside of students' neighborhoods. The contribution of this paper is to show that a particular feature of the assignment mechanism having to do with how school seats are processed by the assignment algorithm – the precedence order – plays a central role in resolution of the tension between these two points of view.

In Boston, we demonstrated that the precedence order has quantitative impacts almost as large as changes in neighborhood priority policy, and is therefore an important lever for achieving distributional objectives. Even though explicit implementations of precedence have not been part of prior school choice policy discussions (with the exception of those at BPS, where the current paper has entered the discussion), it is clear that they should accompany debates about priorities. Precedence order plays a particularly central role in Boston, but it also seems likely that this feature of assignment plays an important role in other priority-based assignment problems where priorities depend on particular slots.

Finally, it is worth noting that our paper uses market design techniques and analysis to show how to achieve various policy objectives. We do not take a stand on the optimal priority or precedence policies. Further analysis might investigate the optimal fraction of neighborhood students in a choice plan.

A Appendix

A.1 Preliminaries for Proposition 1

For a school a^* and a slot $s^* \in S^{a^*}$ of school a^*, suppose that s^* is an open slot under priority structure π, and is a walk-zone slot under priority $\tilde{\pi}$. Suppose furthermore that $\pi^s = \tilde{\pi}^s$ for all slots $s \in S^{a^*}$ other than a^*. Let C^{a^*} and $\tilde{C}^{a^*}$ respectively be the choice functions for a^* induced by the priorities π and $\tilde{\pi}$, under (fixed) precedence order $\rhd^{a^*}$.

Lemma 1. *For any set of students $\bar{I} \subseteq I$:*

1. *All students in the walk-zone of a^* that are chosen from $\bar{I}$ under choice function C^{a^*} are chosen under choice function $\tilde{C}^{a^*}$ (i.e. $[(C^{a^*}(\bar{I})) \cap I_{a^*}] \subseteq [(\tilde{C}^{a^*}(\bar{I})) \cap I_{a^*}]$).*
2. *All students not in the walk-zone of a^* that are from $\bar{I}$ chosen under choice function $\tilde{C}^{a^*}$ are chosen under choice function C^{a^*} (i.e. $[(\tilde{C}^{a^*}(\bar{I})) \cap (I \setminus I_{a^*})] \subseteq [(C^{a^*}(\bar{I})) \cap (I \setminus I_{a^*})]$).*

Proof. We proceed by induction on the number q_{a^*} of slots at a^*. The base case $q_{a^*} = 1$ is immediate, as then $S^{a^*} = \{s^*\}$ and $C^{a^*}(\bar{I}) \neq \tilde{C}^{a^*}(\bar{I})$ if and only if a walk-zone student of a^* is assigned to s^* under $\tilde{C}$, but a non-walk-zone student is assigned to s^* under C, that is, if $\tilde{C}^{a^*}(\bar{I}) \subseteq I_{a^*}$ while $C^{a^*}(\bar{I}) \subseteq I \setminus I_{a^*}$. It follows immediately from this observation that $[(C^{a^*}(\bar{I})) \cap I_{a^*}] \subseteq [(\tilde{C}^{a^*}(\bar{I})) \cap I_{a^*}]$ and $[(\tilde{C}^{a^*}(\bar{I})) \cap (I \setminus I_{a^*})] \subseteq [(C^{a^*}(\bar{I})) \cap (I \setminus I_{a^*})]$.

Now, given the result for the base case $q_{a^*} = 1$, we suppose that the result holds for all $q_{a^*} < \ell$ for some $\ell \geq 1$; we show that this implies the result for $q_{a^*} = \ell$. We suppose that $q_{a^*} = \ell$, and let $\bar{s} \in S^{a^*}$ be the slot which is minimal (i.e., processed last) under the precedence order $\rhd^{a^*}$. A student eligible for one type of slot is also eligible for the other, and hence $\bar{s}$ is either full in both cases or empty in both cases. Moreover, the result follows directly from the inductive hypothesis in the case if no student is assigned to $\bar{s}$ (under either priority structure); hence, we assume that

$$|C^{a^*}(\bar{I})| = |\tilde{C}^{a^*}(\bar{I})| = q_{a^*} = \ell. \tag{1}$$

If $\bar{s} = s^*$, then the result follows just as in the base case: It is clear from the algorithms defining C^{a^*} and $\tilde{C}^{a^*}$ that the same students are assigned to slots $s \rhd^{a^*} s^* = \bar{s}$ in the computations of $C^{a^*}(\bar{I})$ and $\tilde{C}^{a^*}(\bar{I})$, as those slots' priorities and relative precedence ordering fixed. Thus, as in the base case, $C^{a^*}(\bar{I}) \neq \tilde{C}^{a^*}(\bar{I})$ if and only if a walk-zone student of a^* is assigned to s^* under $\tilde{C}$, but a non-walk-zone student is assigned to s^* under C.

If $\bar{s} \neq s^*$, then $s^* \rhd^{a^*} \bar{s}$. We let $J \subseteq \bar{I}$ be the set of students assigned to slots in $S^{a^*} \setminus \{\bar{s}\}$ in the computation of $C^{a^*}(\bar{I})$, and let $\tilde{J} \subseteq \bar{I}$ be the set of students assigned to slots in $S^{a^*} \setminus \{\bar{s}\}$ in the computation of $\tilde{C}^{a^*}(\bar{I})$. The inductive hypothesis, in the case $q_{a^*} = \ell - 1$, implies

$$[J \cap I_{a^*}] \subseteq [\tilde{J} \cap I_{a^*}], \tag{2}$$

$$[\tilde{J} \cap (I \setminus I_{a^*})] \subseteq [J \cap (I \setminus I_{a^*})], \tag{3}$$

as the first q_{a^*} slots of a^* can be treated as a school with slot-set $S^{a^*} \setminus \{\bar{s}\}$ (under the precedence order induced by $\rhd^{a^*}$).

If we have equality in (2) and (3),[9] then the set of students available to be assigned to $\bar{s}$ in the computation of $C^{a^*}(\bar{I})$ is the same as in the computation of $\tilde{C}^{a^*}(\bar{I})$. Thus, since $\pi^{\bar{s}} = \tilde{\pi}^{\bar{s}}$ by assumption, we have $C^{a^*}(\bar{I}) = \tilde{C}^{a^*}(\bar{I})$; hence, the desired result follows trivially.

If instead the inclusions in (2) and (3) are strict, then there is some student $i \in [\tilde{J} \cap I_{a^*}] \setminus [J \cap I_{a^*}]$ who is in the walk-zone of a^* and is assigned to a slot $s \rhd^{a^*} \bar{s}$ in the computation of $\tilde{C}^{a^*}(\bar{I})$ but is not assigned to such a slot in the computation of $C^{a^*}(\bar{I})$. We let $\bar{i}$ be the student in $[\tilde{J} \cap I_{a^*}] \setminus [J \cap I_{a^*}]$ ranked highest under π^o; by construction, $\bar{i}$ must be the π^o-maximal student in $[\bar{I} \setminus J] \cap I_{a^*}$. Thus:

- If $\bar{s}$ is assigned a walk-zone student of a^* in the computation of $C^{a^*}(\bar{I})$, then that student must be $\bar{i}$. Then, $C^{a^*}(\bar{I}) = J \cup \{\bar{i}\}$; hence,

$$[(C^{a^*}(\bar{I})) \cap I_{a^*}] = [(J \cup \{\bar{i}\}) \cap I_{a^*}] \subseteq [\tilde{J} \cap I_{a^*}] \cup \{\bar{i}\},$$

  where the inequality follows from (2). Since $\bar{i} \in [\tilde{J} \cap I_{a^*}] \subseteq [(\tilde{C}^{a^*}(\bar{I})) \cap I_{a^*}]$, it follows that

$$[(C^{a^*}(\bar{I})) \cap I_{a^*}] \subseteq [(\tilde{C}^{a^*}(\bar{I})) \cap I_{a^*}]. \tag{4}$$

- If $\bar{s}$ is assigned a student not in the walk-zone of a^* in the computation of $C^{a^*}(\bar{I})$, then (2) directly implies (4).

This completes the first half of the induction.

Likewise, if the inclusions in (2) and (3) are strict, then there is some student $i \in [J \cap (I \setminus I_{a^*})] \setminus [\tilde{J} \cap (I \setminus I_{a^*})]$ who is not in the walk-zone of a^*, is assigned to a slot $s \rhd^{a^*} \bar{s}$ in the computation of $C^{a^*}(\bar{I})$, and is not assigned to such a slot in the computation of $\tilde{C}^{a^*}(\bar{I})$. We let $\hat{i}$ be the student in $[J \cap (I \setminus I_{a^*})] \setminus [\tilde{J} \cap (I \setminus I_{a^*})]$ ranked highest under π^o; by construction, $\hat{i}$ must be the π^o-maximal student in $[\bar{I} \setminus \tilde{J}] \cap (I \setminus I_{a^*})$. Thus:

- If $\bar{s}$ is assigned a student not in the walk zone of a^* in the computation of $\tilde{C}^{a^*}(\bar{I})$, then that student must be $\hat{i}$. Then, $\tilde{C}^{a^*}(\bar{I}) = \tilde{J} \cup \{\hat{i}\}$; hence,

$$[(\tilde{C}^{a^*}(\bar{I})) \cap (I \setminus I_{a^*})] = [(\tilde{J} \cup \{\hat{i}\}) \cap (I \setminus I_{a^*})] \subseteq [J \cap (I \setminus I_{a^*})] \cup \{\hat{i}\},$$

  where the inequality follows from (3). Since $\hat{i} \in [J \cap (I \setminus I_{a^*})] \subseteq [(C^{a^*}(\bar{I})) \cap (I \setminus I_{a^*})]$, it follows that

$$[(\tilde{C}^{a^*}(\bar{I})) \cap (I \setminus I_{a^*})] \subseteq [(C^{a^*}(\bar{I})) \cap (I \setminus I_{a^*})]. \tag{5}$$

- If $\bar{s}$ is assigned a walk-zone student of a^* in the computation of $\tilde{C}^{a^*}(\bar{I})$, then (3) directly implies (5).

These observations complete the second half of the induction. $\square$

[9] As $|J| = |\tilde{J}|$ by (1), equality holds in one of (2) and (3) if and only if it holds for *both* inclusions (2) and (3).

A.2 Preliminaries for Proposition 2

For a school a^* and a slot $s_w^* \in S^{a^*}$ of school a^*, suppose that s_w^* is a walk-zone slot. Suppose that precedence order $\tilde{\triangleright}$ is obtained from $\triangleright$ by swapping the positions of s_w^* and some open slot $s_o^* \in S^{a^*}$ that is below s_w^* in the order $\triangleright^{a^*}$ (i.e. $s_w^* \triangleright^{a^*} s_o^*$), and leaving the positions of all other slots unchanged. Let C^{a^*} and $\tilde{C}^{a^*}$ respectively be the choice functions for a^* induced by the precedence orders $\triangleright$ and $\tilde{\triangleright}$, under (fixed) slot priorities π^s ($s \in S^{a^*}$).

Lemma 2. *For any set of students $\bar{I} \subseteq I$:*

1. *All students in the walk-zone of a^* that are chosen from $\bar{I}$ under choice function C^{a^*} are chosen under choice function $\tilde{C}^{a^*}$ (i.e. $[(C^{a^*}(\bar{I})) \cap I_{a^*}] \subseteq [(\tilde{C}^{a^*}(\bar{I})) \cap I_{a^*}]$).*
2. *All students not in the walk-zone of a^* that are from $\bar{I}$ chosen under choice function $\tilde{C}^{a^*}$ are chosen under choice function C^{a^*} (i.e. $[(\tilde{C}^{a^*}(\bar{I})) \cap (I \setminus I_{a^*})] \subseteq [(C^{a^*}(\bar{I})) \cap (I \setminus I_{a^*})]$).*

Proof. We proceed by induction on the number q_{a^*} of slots at a^*.

First, we prove the base case $q_{a^*} = 2$.[10] We denote by $i_{s_w^*}$ and $i_{s_o^*}$ (resp. $\tilde{i}_{s_w^*}$ and $\tilde{i}_{s_o^*}$) the students respectively assigned to slots s_w^* and s_o^* in the computation of $C^{a^*}(\bar{I})$ (resp. $\tilde{C}^{a^*}(\bar{I})$). Now:

- If $\{i_{s_w^*}, i_{s_o^*}\} \subset I_{a^*}$, then the ordering under π^o must take the form

  $$\pi^o : i_{s_w^*} \succ i_{s_o^*} \succ \cdots ,$$

  as otherwise some student $i \neq i_{s_w^*}$ would have higher rank than $i_{s_o^*}$ under π^o, and would thus have higher claim than $i_{s_o^*}$ for (open) slot s_o^* under precedence order $\triangleright^{a^*}$. But then, $i_{s_w^*}$ is the π^o-maximal student in $\bar{I}$ and $i_{s_o^*}$ is the π^o-maximal walk-zone student in $\bar{I} \setminus \{i_{s_w^*}\}$; hence, we must have $\tilde{i}_{s_o^*} = i_{s_w^*}$ and $\tilde{i}_{s_w^*} = i_{s_o^*}$, so that $\tilde{C}^{a^*}(\bar{I}) = C^{a^*}(\bar{I})$.

- If $\{i_{s_w^*}, i_{s_o^*}\} \subset (I \setminus I_{a^*})$, then $\bar{I}$ contains no students in the walk-zone of a^* (i.e. $\bar{I} \cap I_{a^*} = \emptyset$) and $i_{s_w^*}$ and $i_{s_o^*}$ are then just the π^o-maximal non-walk-zone students in $\bar{I}$. In this case, we find that $\tilde{i}_{s_o^*} = i_{s_w^*}$ and $\tilde{i}_{s_w^*} = i_{s_o^*}$; hence, $\tilde{C}^{a^*}(\bar{I}) = C^{a^*}(\bar{I})$.

- If $i_{s_w^*} \in I_{a^*}$ and $i_{s_o^*} \in (I \setminus I_{a^*})$, then $i_{s_w^*}$ is the π^o-maximal walk-zone student of a^* in $\bar{I}$. If $i_{s_w^*}$ is also π^o-maximal among *all* students in $\bar{I}$, then we have $\tilde{i}_{s_o^*} = i_{s_w^*}$. Moreover, in this case either $\tilde{i}_{s_w^*} \in I_{a^*}$, or $\tilde{i}_{s_w^*}$ is the only walk-zone student of a^* in $\bar{I}$, so that $\tilde{i}_{s_w^*} = i_{s_o^*}$.

  Alternatively, if $i_{s_w^*}$ is not π^o-maximal among all students in $\bar{I}$, then $i_{s_o^*}$ must be π^o-maximal among all students in $\bar{I}$, so that $\tilde{i}_{s_o^*} = i_{s_o^*}$ and $\tilde{i}_{s_w^*} = i_{s_w^*}$. In either case, we have $[(C^{a^*}(\bar{I})) \cap I_{a^*}] = \{i_{s_w^*}\} \subseteq [(\tilde{C}^{a^*}(\bar{I})) \cap I_{a^*}]$. Additionally, we have $[(\tilde{C}^{a^*}(\bar{I})) \cap (I \setminus I_{a^*})] \subseteq \{i_{s_o^*}\} = [(C^{a^*}(\bar{I})) \cap (I \setminus I_{a^*})]$.

- We cannot have $i_{s_w^*} \in (I \setminus I_{a^*})$ and $i_{s_o^*} \in I_{a^*}$, as s_w^* is a walk-zone slot (and thus gives all students in I_{a^*} higher priority than students in $I \setminus I_{a^*}$) and $s_w^* \triangleright^{a^*} s_o^*$.

[10] Note that the setup requires at least two distinct slots of a^*, so $q_{a^*} = 2$ *a priori*.

The preceding four cases are exhaustive and the desired result holds in each; thus, we have the base case.

Now, given the result for the base case $q_{a^*} = 2$, we suppose that the result holds for all $q_{a^*} < \ell$ for some $\ell \geq 2$; we show that this implies the result for $q_{a^*} = \ell$. We observe that it suffices to show the result in the case that s_w^* and s_o^* are *adjacent* under $\rhd^{a^*}$; the result for general positions with $s_w^* \rhd^{a^*} s_o^*$ follows from the adjacency case upon a sequence of adjacent-slot swaps. Thus, we suppose that s_w^* and s_o^* are adjacent under $\rhd^{a^*}$, with $s_w^* \rhd^{a^*} s_o^*$, and suppose that $q_{a^*} = \ell$. We let $\bar{s} \in S^{a^*}$ be the slot which is minimal under the precedence order $\rhd^{a^*}$. A student eligible for one type of slot is also eligible for the other, and hence $\bar{s}$ is either full in both cases or empty in both cases. Moreover, the result follows directly from the inductive hypothesis in the case if no student is assigned to $\bar{s}$ (under either priority structure); hence, we assume that

$$|C^{a^*}(\bar{I})| = |\tilde{C}^{a^*}(\bar{I})| = q_{a^*} = \ell. \tag{6}$$

If $\bar{s} = s_o^*$, then the result follows just as in the base case, as it is clear from the algorithms defining C^{a^*} and $\tilde{C}^{a^*}$ that the same students are assigned to slots $s \rhd^{a^*} s_w^* \rhd^{a^*} s_o^* = \bar{s}$ in the computations of $C^{a^*}(\bar{I})$ and $\tilde{C}^{a^*}(\bar{I})$.

If $\bar{s} \neq s_o^*$, then $s_w^* \rhd^{a^*} s_o^* \rhd^{a^*} \bar{s}$. We let $J \subseteq \bar{I}$ be the set of students assigned to slots in $S^{a^*} \setminus \{\bar{s}\}$ in the computation of $C^{a^*}(\bar{I})$, and let $\tilde{J} \subseteq \bar{I}$ be the set of students assigned to slots in $S^{a^*} \setminus \{\bar{s}\}$ in the computation of $\tilde{C}^{a^*}(\bar{I})$. The inductive hypothesis, in the case $q_{a^*} = \ell - 1$, implies

$$[J \cap I_{a^*}] \subseteq [\tilde{J} \cap I_{a^*}], \tag{7}$$

$$[\tilde{J} \cap (I \setminus I_{a^*})] \subseteq [J \cap (I \setminus I_{a^*})], \tag{8}$$

as the first q_{a^*} slots of a^* can be treated as a school with slot-set $S^{a^*} \setminus \{\bar{s}\}$ (under the precedence order induced by $\rhd^{a^*}$).

If we have equality in (7) and (8),[11] then the set of students available to be assigned to $\bar{s}$ in the computation of $C^{a^*}(\bar{I})$ is the same as in the computation of $\tilde{C}^{a^*}(\bar{I})$; the desired result then follows trivially.

If instead the inclusions in (7) and (8) are strict, then there is some student $i \in [\tilde{J} \cap I_{a^*}] \setminus [J \cap I_{a^*}]$ who is in the walk-zone of a^* and is assigned to a slot $s \rhd^{a^*} \bar{s}$ in the computation of $\tilde{C}^{a^*}(\bar{I})$ but is not assigned to such a slot in the computation of $C^{a^*}(\bar{I})$. We let $\bar{i}$ be the student in $[\tilde{J} \cap I_{a^*}] \setminus [J \cap I_{a^*}]$ ranked highest under π^o; by construction, $\bar{i}$ must be the π^o-maximal student in $[\bar{I} \setminus J] \cap I_{a^*}$. Thus:

- If $\bar{s}$ is assigned a walk-zone student of a^* in the computation of $C^{a^*}(\bar{I})$, then that student must be $\bar{i}$. Then, $C^{a^*}(\bar{I}) = J \cup \{\bar{i}\}$; hence,

$$[(C^{a^*}(\bar{I})) \cap I_{a^*}] = [(J \cup \{\bar{i}\}) \cap I_{a^*}] \subseteq [\tilde{J} \cap I_{a^*}] \cup \{\bar{i}\},$$

where the inequality follows from (7). Since $\bar{i} \in [\tilde{J} \cap I_{a^*}] \subseteq [(\tilde{C}^{a^*}(\bar{I})) \cap I_{a^*}]$, it follows that

$$[(C^{a^*}(\bar{I})) \cap I_{a^*}] \subseteq [(\tilde{C}^{a^*}(\bar{I})) \cap I_{a^*}]. \tag{9}$$

[11] As $|J| = |\tilde{J}|$ by (6), equality holds in one of (7) or (8) if and only if it holds for *both* (7) and (8).

- If $\bar{s}$ is assigned a student not in the walk-zone of a^* in the computation of $C^{a^*}(\bar{I})$, then (7) directly implies (9).

Thus, we have completed the first half of the induction.

Likewise, if the inclusions in (7) and (8) are strict, then there is some student $i \in [J \cap (I \setminus I_{a^*})] \setminus [\tilde{J} \cap (I \setminus I_{a^*})]$ who is not in the walk-zone of a^*, is assigned to a slot $s \rhd^{a^*} \bar{s}$ in the computation of $C^{a^*}(\bar{I})$, and is not assigned to such a slot in the computation of $\tilde{C}^{a^*}(\bar{I})$. We let $\hat{\imath}$ be the student in $[J \cap (I \setminus I_{a^*})] \setminus [\tilde{J} \cap (I \setminus I_{a^*})]$ ranked highest under π^o; by construction, $\hat{\imath}$ must be the π^o-maximal student in $[\bar{I} \setminus \tilde{J}] \cap (I \setminus I_{a^*})$. Thus:

- If $\bar{s}$ is assigned a student not in the walk zone of a^* in the computation of $\tilde{C}^{a^*}(\bar{I})$, then that student must be $\hat{\imath}$. Then, $\tilde{C}^{a^*}(\bar{I}) = \tilde{J} \cup \{\hat{\imath}\}$; hence,

$$[(\tilde{C}^{a^*}(\bar{I})) \cap (I \setminus I_{a^*})] = [(\tilde{J} \cup \{\hat{\imath}\}) \cap (I \setminus I_{a^*})] \subseteq [J \cap (I \setminus I_{a^*})] \cup \{\hat{\imath}\},$$

  where the inequality follows from (8). Since $\hat{\imath} \in [J \cap (I \setminus I_{a^*})] \subseteq [(C^{a^*}(\bar{I})) \cap (I \setminus I_{a^*})]$, it follows that

$$[(\tilde{C}^{a^*}(\bar{I})) \cap (I \setminus I_{a^*})] \subseteq [(C^{a^*}(\bar{I})) \cap (I \setminus I_{a^*})]. \tag{10}$$

- If $\bar{s}$ is assigned a walk-zone student of a^* in the computation of $C^{a^*}(\bar{I})$, then (8) directly implies (10).

These observations complete the second half of the induction. □

A.3 Proof of Propositions 1 and 2

Definition. In the *cumulative offer process* under choice functions $\bar{C}$, students propose contracts to schools in a sequence of steps $\ell = 1, 2, \ldots$:

Step 1. Some student $i^1 \in I$ proposes to his favorite school a^1. Set $\bar{A}^2_{a^1} = \{i^1\}$, and set $\bar{A}^2_a = \emptyset$ for each $a \neq a^1$; these are the sets of students *available* to schools at the beginning of Step 2. Each school $a \in A$ *holds* $\bar{C}^a(\bar{A}^2_a)$ and *rejects* all other students in $\bar{A}^2_a$.

Step ℓ. Some student $i^\ell \in I$ not currently held by any school proposes to his most-preferred school that has not yet rejected him, a^ℓ. Set $\bar{A}^{\ell+1}_{a^\ell} = \bar{A}^\ell_{a^\ell} \cup \{i^\ell\}$, and set $\bar{A}^{\ell+1}_a = \bar{A}^\ell_a$ for each $a \neq a^\ell$. Each school $a \in A$ holds $\bar{C}^a(\bar{A}^{\ell+1}_a)$ and rejects all other students in $\bar{A}^{\ell+1}_a$.

If at any Step $\ell+1$ no student is able to propose—that is, if all students not on hold have proposed to all schools they find acceptable—then the process terminates. The *outcome of the cumulative offer process* is the matching $\bar{\mu}$ which assigns each school $a \in A$ the students it holds at the end of the last step before termination: $\bar{\mu}_a = \bar{C}^a(\bar{A}^{\ell+1}_a)$.

In our context, the cumulative offer process outcome is independent of the proposal order and is equal to the outcome of the student-optimal stable mechanism (see (Kominers and Sönmez 2012)).

We consider a modification of the cumulative offer process in which some students may be rejected as soon as they propose.

Definition. For sets of students $R_a \subseteq I$ $(a \in A)$, let the *cumulative offer process with premature rejection* under choice functions $\bar{C}$ be the following algorithm in which students propose contracts to schools in a sequence of steps $\ell = 1, 2, \ldots$:

Step 1. Some student $i^1 \in I$ proposes to his favorite school a^1. Set $\bar{E}^2_{a^1} = \{i^1\}$, and set $\bar{E}^2_a = \emptyset$ for each $a \neq a^1$; these are the sets of students *available* to schools at the beginning of Step 2. Each school $a \in A$ *holds* $[(\bar{C}^a(\bar{E}^2_a)) \setminus R_a]$ and *rejects* all other students in $\bar{E}^2_a$.

Step ℓ. Some student $i^\ell \in I$ not currently held by any school proposes to his most-preferred school that has not yet rejected him, a^ℓ. Set $\bar{E}^{\ell+1}_{a^\ell} = \bar{E}^\ell_{a^\ell} \cup \{i^\ell\}$, and set $\bar{E}^{\ell+1}_a = \bar{E}^\ell_a$ for each $a \neq a^\ell$. Each school $a \in A$ holds $[(\bar{C}^a(\bar{E}^{\ell+1}_a)) \setminus R_a]$ and rejects all other students in $\bar{E}^{\ell+1}_a$.

If at any Step $\ell+1$ no student is able to propose—that is, if all students not on hold have proposed to all schools they find acceptable—then the process terminates. The *outcome of the cumulative offer process with premature rejection* is the matching $\bar{\mu}$ which assigns each school $a \in A$ the students it holds at the end of the last step before termination: $\bar{\mu}_a = [(\bar{C}^a(\bar{E}^{\ell+1}_a)) \setminus R_a]$.

Lemma 3. *For each school $a \in A$, let $\bar{R}_a$ be the set of students rejected by a during the cumulative offer process under choice functions $\bar{C}$. For any choice of sets $R_a \subseteq \bar{R}_a$ $(a \in A)$ and any sequence Σ of student$\to$school proposals $\Sigma = \langle (i^1 \to a^1), (i^2 \to a^2), \ldots \rangle$ that can arise in the cumulative offer process under choice functions $\bar{C}$:*

1. *Proposal sequence Σ is a valid proposal sequence in the cumulative offer process with premature rejection under choice functions $\bar{C}$.*

2. *Under proposal sequence Σ, the outcome of the cumulative offer process with premature rejection under choice functions $\bar{C}$, is the same as the outcome of the cumulative offer process under choice functions $\bar{C}$.*

Proof. We denote by A^ℓ_a (resp. E^ℓ_a) the sets of students available to each school $a \in A$ at the start of Step $\ell \geq 2$ of the cumulative offer process under choice functions $\bar{C}$ (resp. the cumulative offer process with premature rejection under choice functions $\bar{C}$).

It is clear that if proposal sequence Σ is used for the first ℓ steps of each process, then $A^{m+1}_a = E^{m+1}_a$ for all $m \leq \ell$. In this case, $\Sigma_{\ell+1} = (i^{\ell+1} \to a^{\ell+1})$ is a valid proposal in Step $\ell+1$ of the cumulative offer process with premature rejection under choice functions $\bar{C}$.[12] And if $\Sigma_{\ell+1} = (i^{\ell+1} \to a^{\ell+1})$ is proposed in Step $\ell+1$ of the cumulative offer process with premature rejection under choice functions $\bar{C}$, we have $E_a^{\ell+2} = E_a^{\ell+1} \cup \{i^{\ell+1}\} = A_a^{\ell+2}$ for each $a \in A$.

Now, Σ_1 is a valid proposal for the *first* step of each process. Inductive application of the preceding observations thus shows that Σ is a valid proposal sequence in the cumulative offer process with premature rejection under choice functions $\bar{C}$. Moreover, under that proposal sequence we have $E_a^{\ell+1} = A_a^{\ell+1}$ for each ℓ and $a \in A$; hence, the eventual outcome of the cumulative offer process with premature rejection must be the same as the outcome of the cumulative offer process. $\square$

Now, we prove Propositions 1 and 2 using a completely parallel argument for the two results.

In the sequel, we assume the setup of either Section A.1 or Section A.2, let $C^a = \tilde{C}^a$ for all schools $a \neq a^*$, and let μ and $\tilde{\mu}$ respectively denote the cumulative offer process outcomes under the choice functions C and $\tilde{C}$. We make use of an *Adjustment Lemma*, which is Lemma 1 for the case of Proposition 1 and Lemma 2 for the case of Proposition 2.

Proposition 6. *Either*

- $\mu_{a^*} = \tilde{\mu}_{a^*}$*—the same students are assigned to* a^* *under* μ *and* $\tilde{\mu}$*—or*
- $\mathfrak{n}_{a^*}(\tilde{\mu}) > \mathfrak{n}_{a^*}(\mu)$*—*$a^*$ *has strictly more walk-zone assignment under* $\tilde{\mu}$ *than under* μ*.*

Proof. We let $R_{a^*} \subseteq I$ be the set of students who are rejected from a^* in *both* the cumulative offer process under the choice function C *and* the cumulative offer process under the choice function $\tilde{C}$, and let $R_a = \emptyset$ for all $a \neq a^*$. By Lemma 3, μ and $\tilde{\mu}$ are the outcomes of the cumulative offer processes with premature rejection under the choice functions C and $\tilde{C}$, respectively.

We now consider the cumulative offer processes with premature rejection under the choice functions C and $\tilde{C}$, with orders of proposal chosen to be identical for the maximal number of steps possible[13]; we let E_a^ℓ and $\tilde{E}_a^\ell$ be the associated sets of effectively available students. If $\mu_a \neq \tilde{\mu}_a$, then there is some Step ℓ' such that $[(C^{a^*}(E_{a^*}^{\ell'+1})) \setminus R_{a^*}] \neq [(\tilde{C}^{a^*}(\tilde{E}_{a^*}^{\ell'+1})) \setminus R_{a^*}]$; we let ℓ be the minimal such ℓ'. We let $n \equiv |I_{a^*} \cap [(C^{a^*}(E_{a^*}^{\ell+1})) \setminus R_{a^*}]|$ be the number of walk-zone students held by a^* at Step ℓ under choice function C^{a^*}, and let $\tilde{n} \equiv |I_{a^*} \cap [(\tilde{C}^{a^*}(\tilde{E}_{a^*}^{\ell+1})) \setminus R_{a^*}]|$ be the number of walk-zone students held by a^* at Step ℓ under choice function $\tilde{C}^{a^*}$.

By our choice of proposal orders and the minimality of ℓ, we know that $E_{a^*}^{\ell+1} = \tilde{E}_{a^*}^{\ell+1}$; hence, $\tilde{C}^{a^*}(E_{a^*}^{\ell+1}) = \tilde{C}^{a^*}(\tilde{E}_{a^*}^{\ell+1})$. As $[C^{a^*}(E_{a^*}^{\ell+1}) \setminus R_{a^*}] \neq [\tilde{C}^{a^*}(\tilde{E}_{a^*}^{\ell+1}) \setminus R_{a^*}]$, we know that

$$C^{a^*}(E_{a^*}^{\ell+1}) \neq \tilde{C}^{a^*}(\tilde{E}_{a^*}^{\ell+1}) = \tilde{C}^{a^*}(E_{a^*}^{\ell+1}) \quad (12)$$

[12] To see this, it suffices to note that $i^{\ell+1}$ is rejected in Step ℓ of the cumulative offer process under choice functions $\bar{C}$; hence,

$$i^{\ell+1} \notin \bar{C}^a(A_a^{\ell+1}) \quad (11)$$

for all $a \in A$. As $A_a^{\ell+1} = E_a^{\ell+1}$ for all all $a \in A$, (11) implies that $i^{\ell+1}$ is rejected in Step ℓ of the cumulative offer process with premature rejection under choice functions $\bar{C}$, and thus is free to propose in Step $\ell+1$.

[13] Such a maximum clearly exists, as the number of steps is bounded above by $|I| \cdot |A|$.

We thus see from the first part of the Adjustment Lemma that there is a student $i \in I_{a^*}$ such that

$$i \notin C^{a^*}(E^{\ell+1}_{a^*}) \quad \text{but} \quad i \in \tilde{C}^{a^*}(E^{\ell+1}_{a^*}) = \tilde{C}^{a^*}(\tilde{E}^{\ell+1}_{a^*});$$

it follows that

$$\tilde{n} > n. \tag{13}$$

We let $\tilde{i}_{\mathrm{m}} \in [I_{a^*} \cap [(\tilde{C}^{a^*}(\tilde{E}^{\ell+1}_{a^*})) \setminus R_{a^*}]]$ be the student in the walk-zone of a^* with the *lowest* ranking under π^o of all students held in Step ℓ of the cumulative offer process with premature rejection under the choice functions $\tilde{C}$.

Claim. *School a^* never rejects student $\tilde{i}_{\mathrm{m}}$ during the running of the cumulative offer process with premature rejection under the choice functions $\tilde{C}$.*

Proof. Since $\tilde{i}_{\mathrm{m}} \notin R_{a^*}$, it suffices to show that $\tilde{i}_{\mathrm{m}}$ is rejected by a^* by Step ℓ of the cumulative offer process with premature rejection under the choice functions C. To see this, we suppose the contrary, that $\tilde{i}_{\mathrm{m}} \in [(C^{a^*}(E^{\ell+1}_{a^*})) \setminus R_{a^*}] \subseteq C^{a^*}(E^{\ell+1}_{a^*})$.

First, we observe that any

$$i \in [I_{a^*} \cap [(\tilde{C}^{a^*}(\tilde{E}^{\ell+1}_{a^*})) \setminus R_{a^*}]] \subseteq \tilde{E}^{\ell+1}_{a^*} = E^{\ell+1}_{a^*}$$

ranked higher than $\tilde{i}_{\mathrm{m}}$ under π^o must be in $C^{a^*}(E^{\ell+1}_{a^*})$: Otherwise, we would have $\tilde{i}_{\mathrm{m}}$ assigned to some slot $s \in S^{a^*}$ in the computation of $C^{a^*}(E^{\ell+1}_{a^*})$, while

1. $i \notin C^{a^*}(E^{\ell+1}_{a^*})$ (by assumption) and
2. $i \pi^s \tilde{i}_{\mathrm{m}}$ (because $i, \tilde{i}_{\mathrm{m}} \in I_{a^*}$ and $i \pi^o \tilde{i}_{\mathrm{m}}$).

We see from the definition of the function C^{a^*} that this cannot happen—$\tilde{i}_{\mathrm{m}}$ cannot be assigned a slot before i.

The preceding observations imply that

$$[I_{a^*} \cap [(\tilde{C}^{a^*}(\tilde{E}^{\ell+1}_{a^*})) \setminus R_{a^*}]] \subseteq [(C^{a^*}(E^{\ell+1}_{a^*})) \setminus R_{a^*}]. \tag{14}$$

Meanwhile, the second part of the Adjustment Lemma shows that $[(I \setminus I_{a^*}) \cap (\tilde{C}^{a^*}(E^{\ell+1}_{a^*}))] \subseteq [(I \setminus I_{a^*}) \cap (C^{a^*}(E^{\ell+1}_{a^*}))]$; hence

$$[(I \setminus I_{a^*}) \cap [(\tilde{C}^{a^*}(\tilde{E}^{\ell+1}_{a^*})) \setminus R_{a^*}]] = [(I \setminus I_{a^*}) \cap [(\tilde{C}^{a^*}(E^{\ell+1}_{a^*})) \setminus R_{a^*}]] \subseteq [(C^{a^*}(E^{\ell+1}_{a^*})) \setminus R_{a^*}]. \tag{15}$$

Combining (14) and (15), we see that $[(\tilde{C}^{a^*}(\tilde{E}^{\ell+1}_{a^*})) \setminus R_{a^*}] \subseteq [(C^{a^*}(E^{\ell+1}_{a^*})) \setminus R_{a^*}]$, so that we must have

$$[(\tilde{C}^{a^*}(\tilde{E}^{\ell+1}_{a^*})) \setminus R_{a^*}] \subsetneq [(C^{a^*}(E^{\ell+1}_{a^*})) \setminus R_{a^*}] \tag{16}$$

since $[(\tilde{C}^{a^*}(\tilde{E}^{\ell+1}_{a^*})) \setminus R_{a^*}] \neq [(C^{a^*}(E^{\ell+1}_{a^*})) \setminus R_{a^*}]$ by (12). But (16) is impossible from the definition of the function $\tilde{C}^{a^*}$, as it would imply that there is

1. some slot $s \in S^{a^*}$ not assigned a student in the computation of $\tilde{C}^{a^*}(\tilde{E}^{\ell+1}_{a^*})$ and

2. some student $i \in [E_{a^*}^{\ell+1} \setminus R_{a^*}] = [\tilde{E}_{a^*}^{\ell+1} \setminus R_{a^*}]$ not assigned to a slot of a^* in the computation of $\tilde{C}^{a^*}(\tilde{E}_{a^*}^{\ell+1})$;

these two condtions cannot hold simultaneously because all slots rank all students as acceptable.

These observations show that the assumption that $\tilde{i}_{\mathrm{m}} \in [(C^{a^*}(E_{a^*}^{\ell+1})) \setminus R_{a^*}]$ leads to a contradiction; hence, we have the claim. $\square$

The preceding claim implies that *all* walk-zone students $i \in [I_{a^*} \cap [(\tilde{C}^{a^*}(\tilde{E}_{a^*}^{\ell+1})) \setminus R_{a^*}]]$ held by a^* by Step ℓ of the cumulative offer process with premature rejection under the choice functions $\tilde{C}$ are held by a^* in all remaining steps of the process, i.e.

$$\tilde{\mu}_{a^*} \supseteq [I_{a^*} \cap [(\tilde{C}^{a^*}(\tilde{E}_{a^*}^{\ell+1})) \setminus R_{a^*}]].$$

Thus, we see that

$$\mathfrak{n}_{a^*}(\tilde{\mu}) \geq \tilde{n}. \tag{17}$$

Now, we let $i_{\mathrm{m}} \in [(I \setminus I_{a^*}) \cap [(C^{a^*}(E_{a^*}^{\ell+1})) \setminus R_{a^*}]]$ be the student not in the walk-zone of a^* with the *lowest* ranking under π^o of all students held in Step ℓ of the cumulative offer process with premature rejection under the choice functions C.

Claim. *School a^* never rejects student i_{m} during the running of the cumulative offer process with premature rejection under the choice functions C.*

Proof. The argument is completely analogous to that used to prove the preceding claim.

Since $i_{\mathrm{m}} \notin R_{a^*}$, it suffices to show that i_{m} is rejected by a^* by Step ℓ of the cumulative offer process with premature rejection under the choice functions $\tilde{C}$. To see this, we suppose the contrary, that $i_{\mathrm{m}} \in [(\tilde{C}^{a^*}(\tilde{E}_{a^*}^{\ell+1})) \setminus R_{a^*}] \subseteq \tilde{C}^{a^*}(\tilde{E}_{a^*}^{\ell+1})$.

First, we observe that any

$$i \in [(I \setminus I_{a^*}) \cap [(C^{a^*}(E_{a^*}^{\ell+1})) \setminus R_{a^*}]] \subseteq E_{a^*}^{\ell+1} = \tilde{E}_{a^*}^{\ell+1}$$

ranked higher than i_{m} under π^o must be in $\tilde{C}^{a^*}(\tilde{E}_{a^*}^{\ell+1})$: Otherwise, we would have i_{m} assigned to some slot $s \in S^{a^*}$ in the computation of $\tilde{C}^{a^*}(\tilde{E}_{a^*}^{\ell+1})$, while

1. $i \notin \tilde{C}^{a^*}(\tilde{E}_{a^*}^{\ell+1})$ (by assumption) and
2. $i \pi^s i_{\mathrm{m}}$ (because $i, i_{\mathrm{m}} \in (I \setminus I_{a^*})$ and $i \pi^o i_{\mathrm{m}}$).

We see from the definition of the function $\tilde{C}^{a^*}$ that this cannot happen—i_{m} cannot be assigned a slot before i.

The preceding observations imply that

$$[(I \setminus I_{a^*}) \cap [(C^{a^*}(E_{a^*}^{\ell+1})) \setminus R_{a^*}]] \subseteq [(\tilde{C}^{a^*}(\tilde{E}_{a^*}^{\ell+1})) \setminus R_{a^*}]. \tag{18}$$

Meanwhile, the first part of the Adjustment Lemma shows that $[I_{a^*} \cap (C^{a^*}(\tilde{E}_{a^*}^{\ell+1}))] \subseteq [I_{a^*} \cap (\tilde{C}^{a^*}(\tilde{E}_{a^*}^{\ell+1}))]$; hence

$$[I_{a^*} \cap [(C^{a^*}(E_{a^*}^{\ell+1})) \setminus R_{a^*}]] = [I_{a^*} \cap [(C^{a^*}(\tilde{E}_{a^*}^{\ell+1})) \setminus R_{a^*}]] \subseteq [(\tilde{C}^{a^*}(\tilde{E}_{a^*}^{\ell+1})) \setminus R_{a^*}]. \tag{19}$$

Combining (18) and (19), we see that $[(C^{a^*}(E_{a^*}^{\ell+1})) \setminus R_{a^*}] \subseteq [(\tilde{C}^{a^*}(\tilde{E}_{a^*}^{\ell+1})) \setminus R_{a^*}]$, so that we must have

$$[(C^{a^*}(E_{a^*}^{\ell+1})) \setminus R_{a^*}] \subsetneq [(\tilde{C}^{a^*}(\tilde{E}_{a^*}^{\ell+1})) \setminus R_{a^*}] \tag{20}$$

since $[(C^{a^*}(E_{a^*}^{\ell+1})) \setminus R_{a^*}] \neq [(\tilde{C}^{a^*}(\tilde{E}_{a^*}^{\ell+1})) \setminus R_{a^*}]$ by (12). But (16) is impossible from the definition of the function C^{a^*}, as it would imply that there is

1. some slot $s \in S^{a^*}$ not assigned a student in the computation of $C^{a^*}(E_{a^*}^{\ell+1})$ and
2. some student $i \in [\tilde{E}_{a^*}^{\ell+1} \setminus R_{a^*}] = [E_{a^*}^{\ell+1} \setminus R_{a^*}]$ not assigned to a slot of a^* in the computation of $C^{a^*}(E_{a^*}^{\ell+1})$;

these two condtions cannot hold simultaneously because all slots rank all students as acceptable.

These observations show that the assumption that $i_{\mathrm{m}} \in [(\tilde{C}^{a^*}(\tilde{E}_{a^*}^{\ell+1})) \setminus R_{a^*}]$ leads to a contradiction; hence, we have the claim. $\square$

The preceding claim implies that *all* non-walk-zone students $i \in [(I \setminus I_{a^*}) \cap [(C^{a^*}(E_{a^*}^{\ell+1})) \setminus R_{a^*}]]$ held by a^* by Step ℓ of the cumulative offer process with premature rejection under the choice functions C are held by a^* in all remaining steps of the process, i.e.

$$\mu_{a^*} \supseteq [(I \setminus I_{a^*}) \cap [(C^{a^*}(E_{a^*}^{\ell+1})) \setminus R_{a^*}]].$$

Thus, as the quota of a^* is fixed at q_{a^*}, we see that

$$n \geq \mathfrak{n}_{a^*}(\mu). \tag{21}$$

Combining (13), (17), and (21), we see that

$$\mathfrak{n}_{a^*}(\tilde{\mu}) \geq \tilde{n} > n \geq \mathfrak{n}_{a^*}(\mu);$$

this proves the result. $\square$

A.4 The Two-School Model

A.4.1 Preliminaries

Matchings μ and $\tilde{\mu}$ are obtained as in Appendix A.3: Either one of the open slots is replaced with a walk-zone slot, or the precedence position of a walk-zone slot is switched with that of a subsequent open slot to obtain $\tilde{C}$ from C, and $\tilde{\mu}$ and μ are, respectively, the associated cumulative offer process outcomes.

Lemma 4. *We have $|\tilde{\mu}_a| = |\mu_a|$ and $|\tilde{\mu}_b| = |\mu_b|$. That is, the number of slots filled at each school is the same under μ as under $\tilde{\mu}$.*

Proof. If *both* of the schools a and b have an empty slot under either matching, stability implies that all students get their first choices under each matching; hence $\tilde{\mu} = \mu$ and the result holds immediately. Likewise, if neither school has an empty slot under either matching, the result holds immediately since then $|\tilde{\mu}_a| = |\mu_a| = |S^a|$ and $|\tilde{\mu}_b| = |\mu_b| = |S^b|$. Hence the only non-trivial case is when, under one of the matchings, one school is full but the other is not.

Without loss of generality, we suppose that under matching μ, school a has an empty slot whereas school b has all its slots full. Then not only does each student who is assigned a slot at school b under matching μ prefer school b to school a, but also there are at least as many students with a first choice of school b as the number of slots at school b. Thus by stability school b must fill all its slots under matching $\tilde{\mu}$ as well; hence, $|\tilde{\mu}_b| = |\mu_b| = |S^b|$. By assumption,

- there are at least as many slots as students, and
- all students find both schools acceptable;

therefore, we see that

$$|\tilde{\mu}_a| = |I| - |\tilde{\mu}_b| = |I| - |\mu_b| = |\mu_a|.$$

This observation completes the proof. □

A.4.2 Proof of Propositions 3 and 4

We prove Propositions 3 and 4 using a completely parallel argument for the two results. We make use of an *Adjustment Proposition*, which is is Proposition 1 for the case of Proposition 3, and Proposition 2 for the case of Proposition 4.

Proposition 7. *There is weakly more walk-zone assignment under $\tilde{\mu}$ than under μ, that is,*

$$\mathfrak{n}_a(\tilde{\mu}) + \mathfrak{n}_b(\tilde{\mu}) \geq \mathfrak{n}_a(\mu) + \mathfrak{n}_b(\mu).$$

Proof. Without loss of generality, we assume the priority structure of school a has changed (i.e. that $a = a^*$ in the setup of Appendix A.3).

If $\tilde{\mu}_a = \mu_a$, then we have

$$\tilde{\mu}_b = I \setminus \tilde{\mu}_a = I \setminus \mu_a = \mu_b,$$

as by assumption

- there are at least as many slots as students, and
- all students find both schools acceptable.

Thus, in this case the result is immediate.

If $\tilde{\mu}_a \neq \mu_a$,

$$|\tilde{\mu}_a \cap I_a| = \mathfrak{n}_a(\tilde{\mu}) > \mathfrak{n}_a(\mu) = |\mu_a \cap I_a|$$

by the Adjustment Proposition. Therefore Lemma 4 implies that

$$|\tilde{\mu}_a \cap (I \setminus I_a)| = |\tilde{\mu}_a| - |\tilde{\mu}_a \cap I_a| < |\mu_a| - |\mu_a \cap I_a| = |\mu_a \cap (I \setminus I_a)|,$$

which in turn implies that

$$|\tilde{\mu}_a \cap I_b| < |\mu_a \cap I_b|$$

as $I \setminus I_a = I_b$ by assumption. Thus, we see that

$$\mathfrak{n}_b(\tilde{\mu}) = |\tilde{\mu}_b \cap I_b| = |I_b| - |\tilde{\mu}_a \cap I_b| > |I_b| - |\mu_a \cap I_b| = |\mu_b \cap I_b| = \mathfrak{n}_b(\mu)$$

as all students (and in particular all students in I_b) are matched under both μ and $\tilde{\mu}$. Hence in this case

$$\mathfrak{n}_a(\tilde{\mu}) + \mathfrak{n}_b(\tilde{\mu}) > \mathfrak{n}_a(\mu) + \mathfrak{n}_b(\mu);$$

this completes the proof. □

A.4.3 Proof of Proposition 5

Let r_a^1 denote the number of students who rank school a as first choice, and let r_b^1 denote the number of students who rank school b as first choice.

We can obtain the outcome of the DA by either the *student proposing deferred acceptance algorithm* or the *cumulative offer process*. We utilize the former in this proof.

By assumption, $|S^a| + |S^b| \geq |I|$. Thus, as each student has a first choice,

$$|S^a| + |S^b| \geq r_a^1 + r_b^1.$$

Hence, either:

1. $|S^a| \geq r_a^1$ and $|S^b| \geq r_b^1$, or
2. $|S^a| > r_a^1$ and $|S^b| < r_b^1$, or
3. $|S^a| < r_a^1$ and $|S^b| > r_b^1$.

In the first case, the student proposing deferred acceptance algorithm terminates in one step and all students receive their first choices under both μ and $\tilde{\mu}$. Thus, the result is immediate in this case.

The analyses of the second and third cases are analogous, so it suffices to consider the case that $|S^a| > r_a^1$ and $|S^b| < r_b^1$.

Claim. *For this case, under both μ and $\tilde{\mu}$,*

- *the number of students receiving their first choices is equal to $|S^b| + r_a^1$, and*
- *the number of students receiving their second choices is equal to $r_b^1 - |S^b|$.*

Proof. We consider the construction of either μ or $\tilde{\mu}$ through the student proposing deferred acceptance algorithm, and observe that school b receives $r_b^1 > |S^b|$ offers in Step 1, holding $|S^b|$ of these while rejecting $r_b^1 - |S^b|$. School a, meanwhile, receives $r_a^1 < |S^a|$ offers and holds all of them. In Step 2, all students rejected by school b apply to school a, bringing the total number of applicants at school a to $r_a^1 + (r_b^1 - |S^b|)$. As $r_a^1 + (r_b^1 - |S^b|) \leq |S^a|$ by assumption, no student is rejected by school a, and the algorithm terminates in Step 2.[14] Hence, under both μ and $\tilde{\mu}$,

- $|S^b|$ students are assigned to school b as their first choice,
- r_a^1 students are assigned to school a as their first choice, and
- $r_b^1 - |S^b|$ students are assigned to school a as their second choice.

These observations show the claim. □

The preceding claim shows the result for the second case; since an analogous argument shows the result for the third case, this completes the proof.

[14] While it is well-known that stability may be in conflict with Pareto efficiency in general school choice environments (cf. Balinski and Sönmez (1999), Ergin (2002), Abdulkadiroğlu and Sönmez (2003), Kesten (2006), and Kesten (2010)), the above argument shows that DA is Pareto efficient in our two-school environment.

Table 1. Difference between the Current Boston Mechanism and Alternative Walk Zone Splits

	Grade K1			Grade K2			Grade 6		
		Difference relative to current BPS			Difference relative to current BPS			Difference relative to current BPS	
	# students	0% Walk	100% Walk	# students	0% Walk	100% Walk	# students	0% Walk	100% Walk
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
2009	1770	46	336	1715	28	343	2348	54	205
		3%	19%		2%	20%		2%	9%
2010	1977	68	392	1902	62	269	2308	41	171
		3%	20%		3%	14%		2%	7%
2011	2071	50	387	1821	90	293	2073	4	225
		2%	19%		5%	16%		0%	11%
2012	2515	88	504	2301	101	403	2057	24	247
		3%	20%		4%	18%		1%	12%
All	8333	252	1619	7739	281	1308	8786	123	848
		3%	19%		4%	17%		1%	10%

Notes. Table reports fraction of applicants whose assignments differ between the mechanism currently employed in Boston and two alternative mechanisms: one with a priority structure without walk-zone priorities at any seats (0% Walk), and the other with a priority structure with walk-zone priorities at all seats (100% Walk).

Table 2. Number of Students Assigned to Walk Zone School

		Priorities = 0% Walk	Priorities = 50% Walk Changing Precedence						Priorities = 100% Walk
	# students		Walk-NonWalk	Actual BPS	Rotating	Compromise	Balanced	NonWalk-Walk	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
					I. Grade K1				
2009-2012	8333	3849	3879	3930	4080	4227	4305	4570	4787
		46.2%	46.5%	47.2%	49.0%	50.7%	51.7%	54.8%	57.4%
					II. Grade K2				
2009-2012	7739	3657	3685	3753	3842	3900	4037	4214	4377
		47.3%	47.6%	48.5%	49.6%	50.4%	52.2%	54.5%	56.6%
					III. Grade 6				
2009-2012	8786	3439	3476	3484	3542	3657	3691	3797	3907
		39.1%	39.6%	39.7%	40.3%	41.6%	42.0%	43.2%	44.5%

Notes. Table reports fraction of applicants assigned to walk-zone schools under several alternative assignment procedures. 0% Walk implements the student-proposing deferred acceptance mechanism with no walk zone priority; 100% implements the student-proposing deferred acceptance mechanism with all slots having walk-zone priority. Columns (3)-(8) hold the 50/50 school seat split fixed. Walk-NonWalk implements the precedence order in which all walk-zone slots are ahead of non-walk zone slots. Actual BPS implements the current BPS system. Rotating implements the precedence ordering alternating between walk-zone and non walk-zone slots. Compromise implements the precedence order in which exactly half of the walk-zone slots come before all non-walk zone slots, which are in turn followed by the half of the walk-zone slots. Balanced implements Rotating, but uses two random numbers for each student, one for walk-zone slots and the other for non-walk zone slots. NonWalk-Walk implements the precedence order in which all non-walk zone slots are ahead of walk-zone slots.

Table 3. Choices Received by Students in 2010-2011

Choice Received	Priorities = 0% Walk	Priorities = 50% Walk Changing Precedence					Priorities = 100% Walk
		Walk-NonWalk	Current BPS	Compromise	Balanced	NonWalk-Walk	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
				I. Grade K1			
1	920	917	923	925	917	921	905
2	234	233	227	237	233	241	234
3	173	175	177	166	183	165	169
4	98	97	94	98	90	95	103
5	56	56	59	60	60	58	60
6	21	20	18	19	23	24	29
7	19	21	20	14	20	18	21
8	11	12	12	11	11	10	12
9	11	11	11	9	6	7	8
10	5	5	5	3	4	6	4
Unassigned or Admin. Assigned	523	524	525	529	524	526	526
				II. Grade K2			
1	870	872	880	881	870	885	888
2	306	306	300	296	310	292	292
3	198	195	192	186	188	189	184
4	83	85	86	79	77	84	83
5	44	42	39	47	40	37	34
6	4	4	5	9	6	7	7
7	4	3	3	2	6	4	2
8	3	3	5	3	5	5	7
9	1	1	1	1	2	2	1
10	1	1	1	1	1	1	1
Unassigned or Admin. Assigned	307	309	309	316	316	315	322
				III. Grade 6			
1	1273	1271	1271	1265	1233	1246	1231
2	397	399	399	395	396	410	419
3	177	177	177	183	198	186	184
4	43	43	43	46	65	52	59
5	19	20	20	21	17	17	16
6	4	3	3	2	2	1	1
7	0	0	0	1		0	0
8	3	3	3	2	1	2	2
9	0	0	0	0		0	0
Unassigned or Admin. Assigned	157	157	157	158	161	159	161

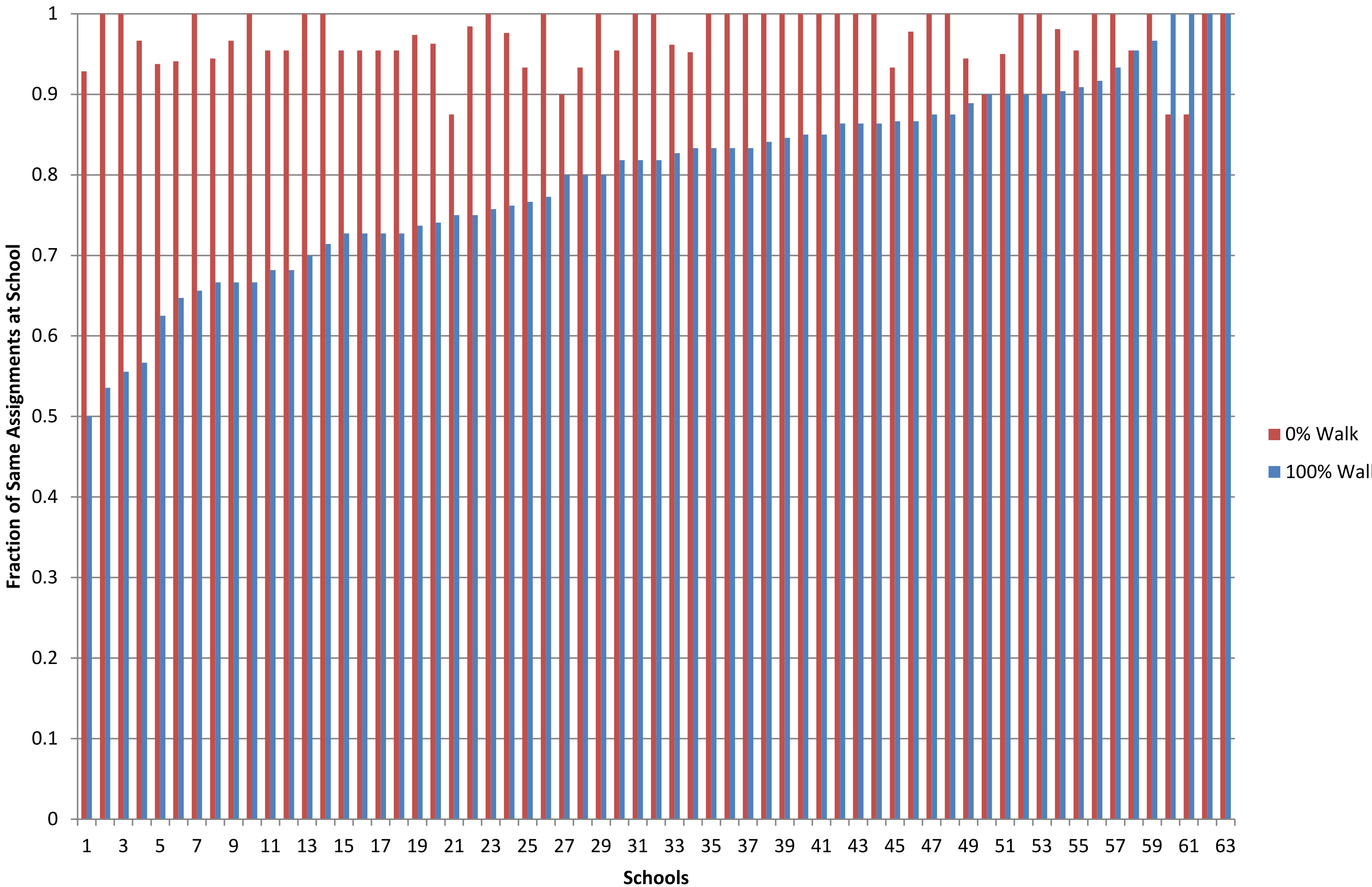
Notes. Table reports the distribution of choice ranks arising under different priority and precedence policies. Unassigned or Adminstrative Assignment means student is not assigned to any of the listed choices; some students will be adminstratively assigned after Round 1.

Table A1. Match with BPS Assignment

year	grade	total	matching students	exact match
32009	K1	1770	1718	97.1
32010	K1	1977	1907	96.5
32011	K1	2071	2044	98.7
32012	K1	2515	2478	98.5
32009	K2	1715	1649	96.2
32010	K2	1902	1855	97.5
32011	K2	1821	1754	96.3
32012	K2	2301	2220	96.5
32009	6	2348	2309	98.3
32010	6	2308	2275	98.6
32011	6	2073	2069	99.8
32012	6	2057	2008	97.6

Notes. Table reports comparison between BPS final assignment and authors' recreation of the assignment for Round 1.

Figure 1. Comparing Assignments under Current BPS Mechanism vs. 0 and 100% Walk Zone Priority for Grade K1 in 2010-2011



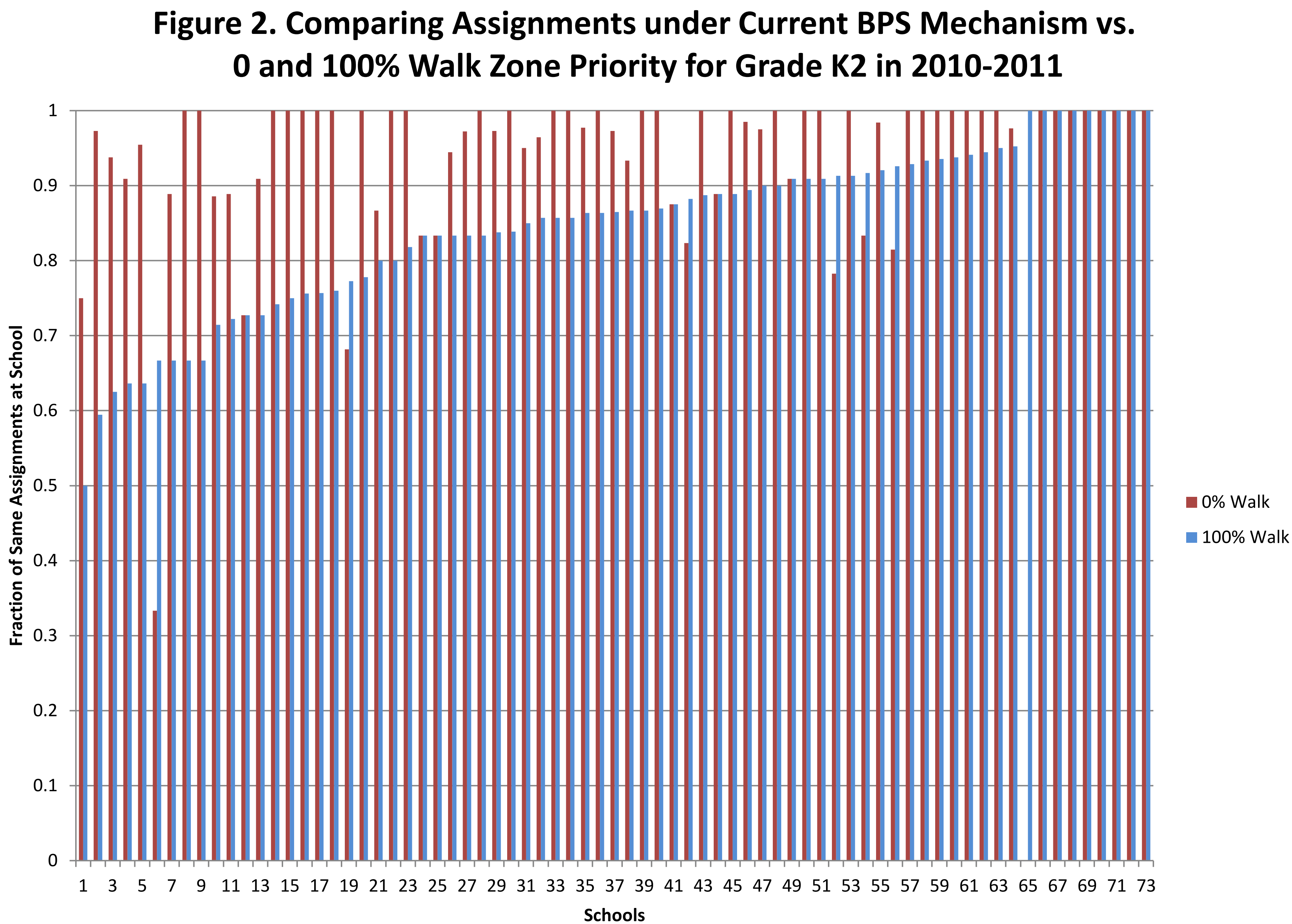
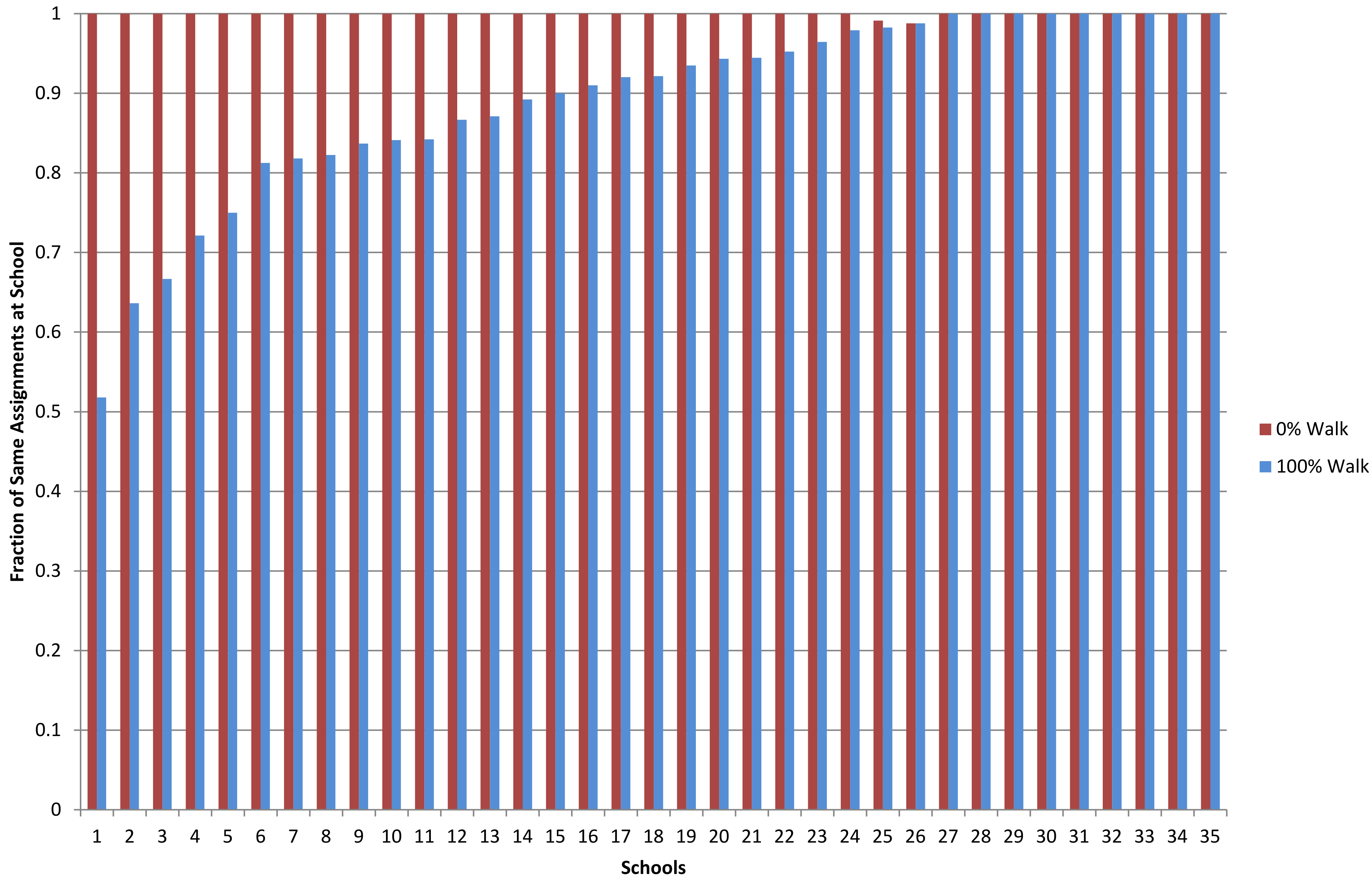


Figure 2. Comparing Assignments under Current BPS Mechanism vs.
0 and 100% Walk Zone Priority for Grade K2 in 2010-2011
Fraction of Same Assignments at School
Schools
0% Walk
100% Walk
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1 3 5 7 9 11 13 15 17 19 21 23 25 27 29 31 33 35 37 39 41 43 45 47 49 51 53 55 57 59 61 63 65 67 69 71 73

Figure 3. Comparing Assignments under Current BPS Mechanism vs. 0% and 100% Walk Zone Priority for Grade 6 in 2010-2011



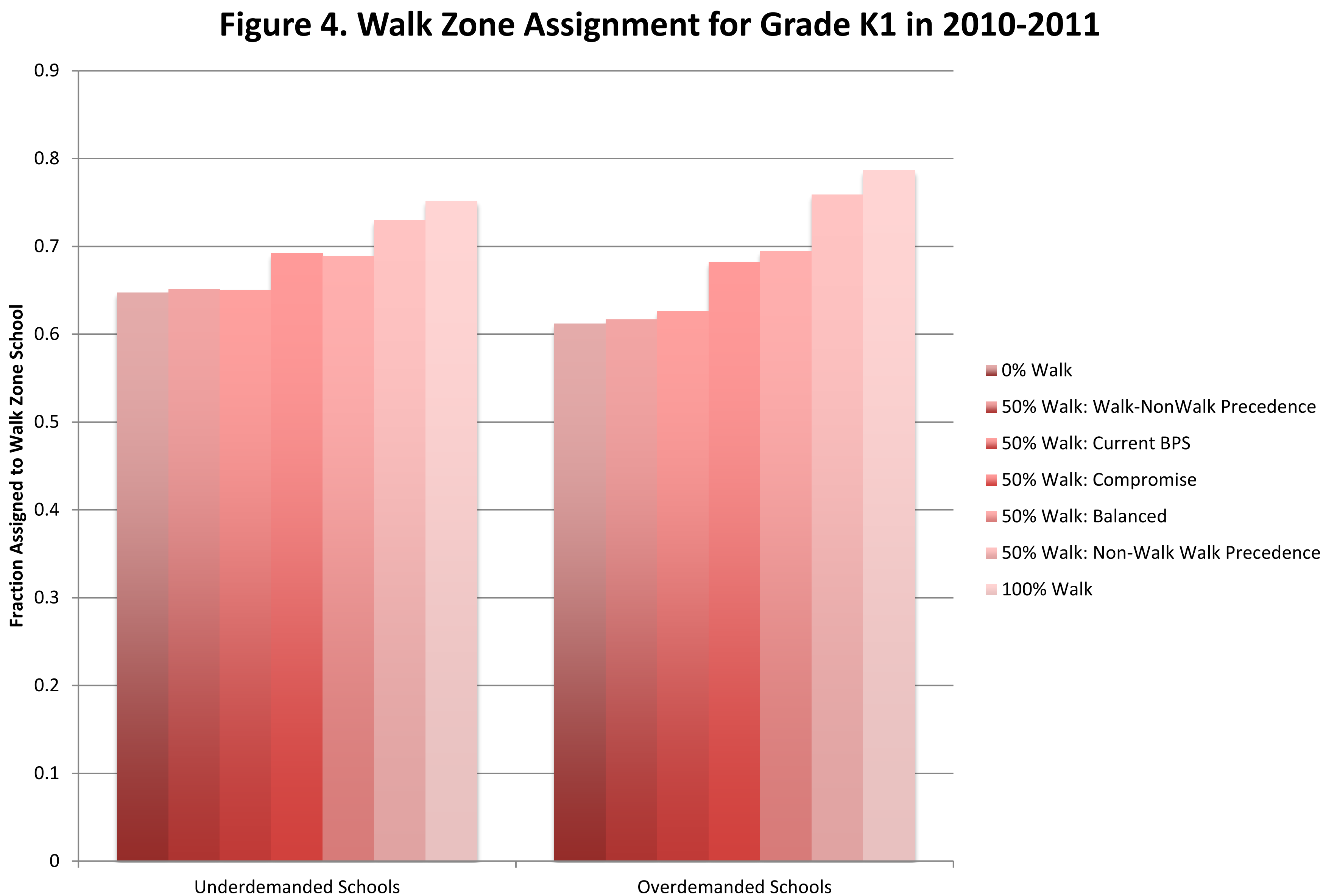


Figure 4. Walk Zone Assignment for Grade K1 in 2010-2011
Fraction Assigned to Walk Zone School
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Underdemanded Schools
Overdemanded Schools
0% Walk
50% Walk: Walk-NonWalk Precedence
50% Walk: Current BPS
50% Walk: Compromise
50% Walk: Balanced
50% Walk: Non-Walk Walk Precedence
100% Walk

Figure 5. Walk Zone Assignment for Grade K2 in 2010-2011

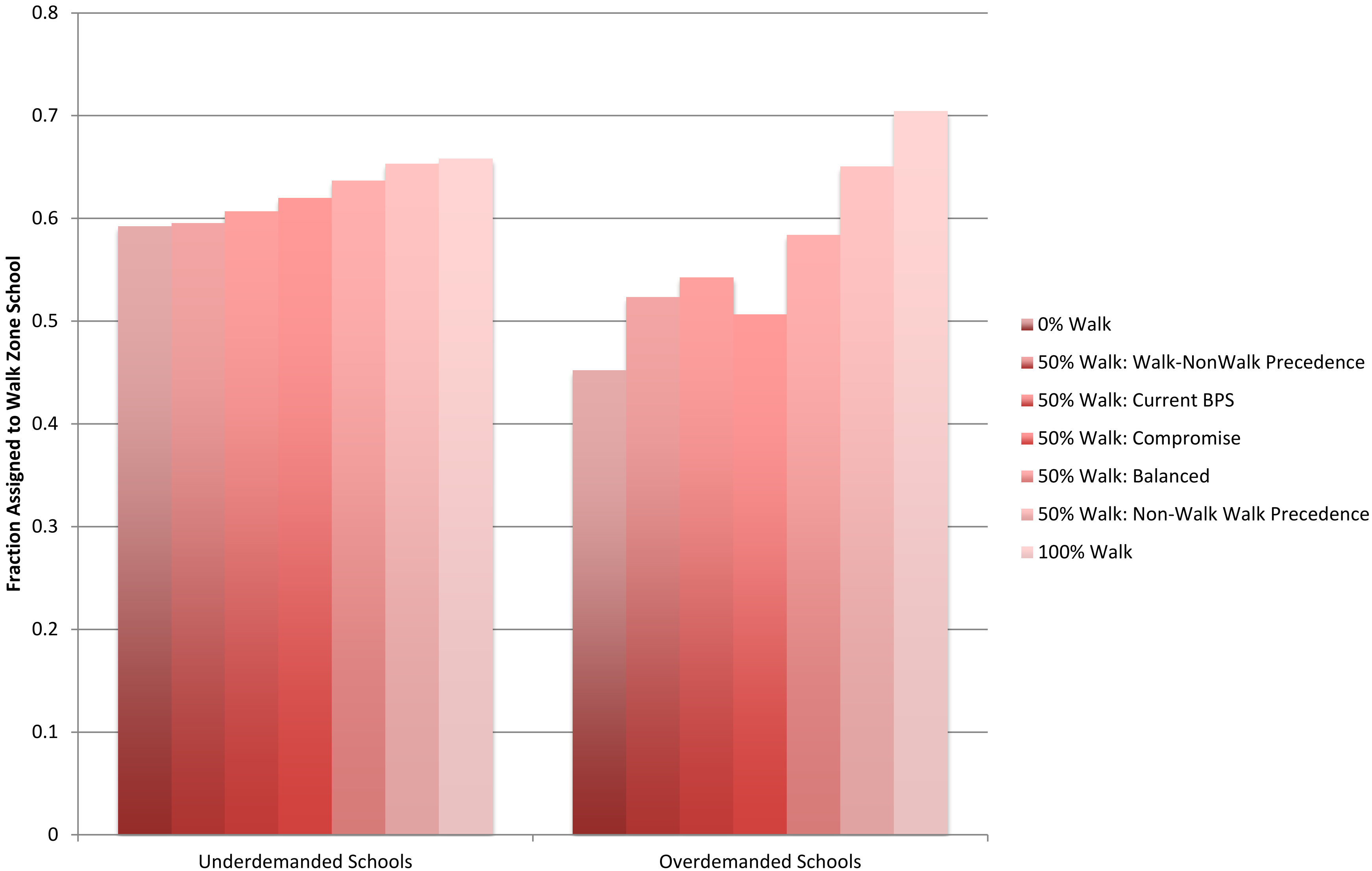
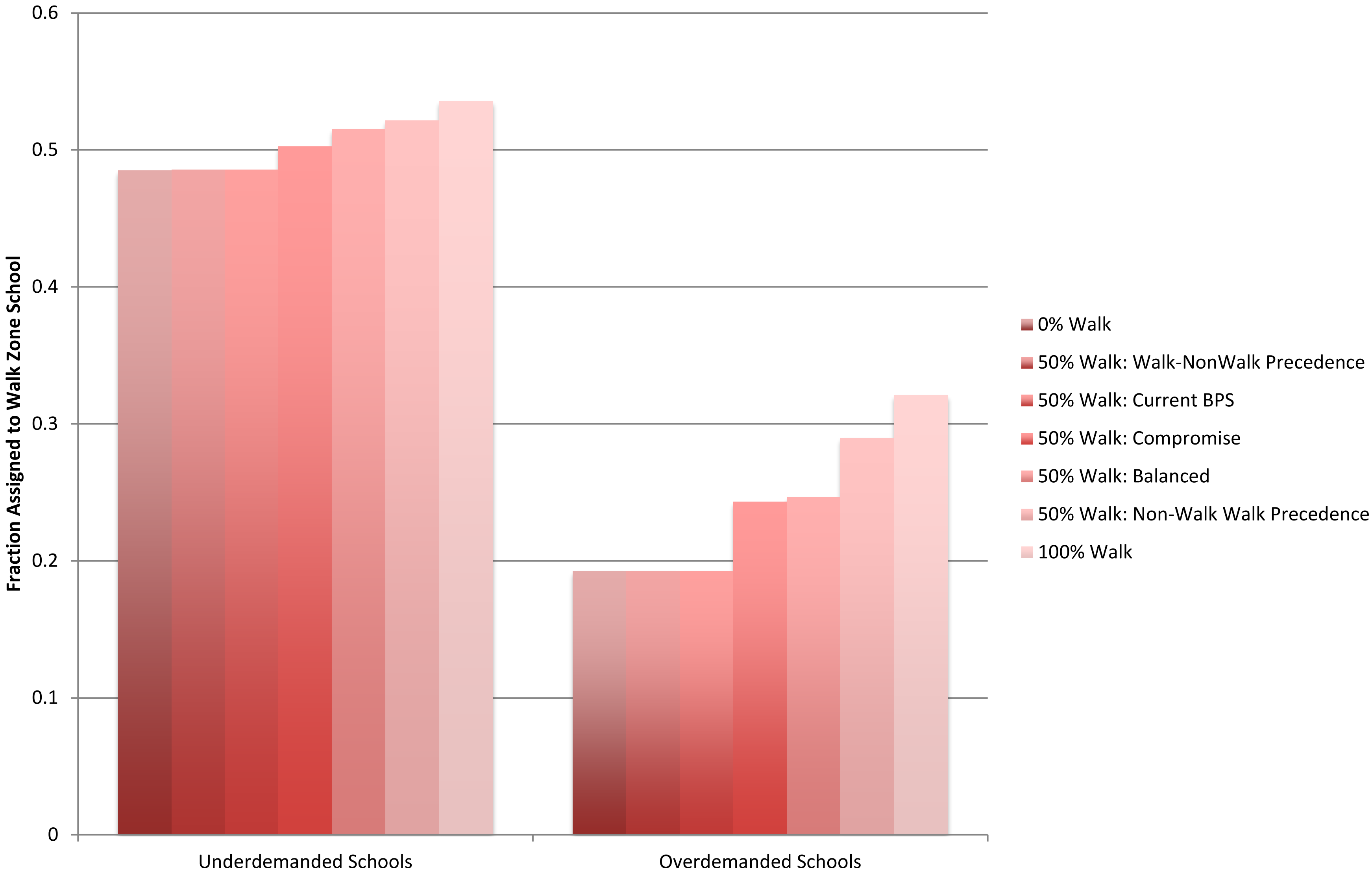


Figure 6. Walk Zone Assignment for Grade 6 in 2010-2011



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